Journal of Mathematical Analysis
ISSN: 2217-3412, URL: www.ilirias.com/jma
Volume 9 Issue 2 (2018), Pages 78-87.

APPLICATION OF GAUSSION RADIAL BASIS FUNCTION WITH CUBIC POLYNOMIAL FOR MODELLING LEAF SURFACE

MOA'ATH N. OQIELAT AND O. OGILAT

Abstract. A new smoothing method based on combining the Gaussion radial basis function with cubic polynomial (GRBFC) to model leaf surface is proposed in this paper. Then a comparison between GRBFC method and Gaussion radial basis function (GRBF) without enhancing a cubic polynomial is given. As such leaf model is important for modelling plant from three dimension scattered data points. Furthermore, this model can be used for visual purposes only such as photosynthesis or for modelling a droplet of water or pesticide movements. For these aims, accuracte leaf model is vital.

Our proposed GRBFC method and GRBF method are validated by a numerical experiment of scattered data and then through a real leaf data sampled using laser scanner from Anthurium leaf. Numerical results shows that the GRBF method produces more realistic and accurate representation of the leaf surface than using GRBFC method.

1. Introduction

Reconstructing a leaf surface from three dimintional scattered data using a digitizing devices is the main aim of the research presented in this paper. This model is important for modelling fluid movements [6, 19, 26] as well as for modelling virtual plant. Modelling virtual plant has been studied by [5, 31, 35, 36].

Recently, Loch [17, 20] used finite element method based on triangulation to represent the surafce of the leaf. Kempthorne et al. [15] used discrete smoothing D^2-splines to reconstruct the surface of the cotton and wheat leaf. The D^2-splines represented as a linear combination of reduced Hsieh-Clought-Tocher method which result in a continous differentiable (C^1) surface. Oqielat et al. [28] applied Hardy's multiquadrics RBF to model the leaf surface. Moreover, Oqielat et al. [24, 25, 26] introduced hybrid interpolation method that joins the RBF and clough-Tocher (CT) method to obtain accurate leaf model. The accuracy of the model is confirmed via numerical experiment of two sets of scattered data. Furthermore, oqielat [29] present a comparsion between many surface fitting methods and found that the best leaf model is achieved using hybrid RBF-CT method. In this paper we combin

2000 *Mathematics Subject Classification.* 93A30, 97Mxx,97N40.
Key words and phrases. Interpolation; virtual leaf; Radial basis function.

Submitted January 2, 2017. Published March 4, 2018.
Communicated by F. Soleymani.

Gaussion radial basis function with cubic polynomial to to achive a continuous leaf surface. More details of some more leaf models can be found in [16, 27, 28, 41, 42].

This research paper consists of five sections. In §2, an intorduction and literature review of the radial basis function and its parameter is given. Moreover, the proposed GRBFC method is given in this section. In §3, the accuracy of the GRBFC method is evaluated via numerical investigation of two data sets taken from Franke [7]. In §4, the application of the GRBFC method to reconstruct the surface of real Frangipani and Anthurium leaf is presented. Lastly, Conclusion is given in §5.

2. Radial Basis Functions Method.

A radial basis function (RBF) method is an capable method for explaining problems in many fields in engineering and sciences [8, 11, 25, 26, 29]. Its originally presented by Hardy [12, 13] to obtain a smooth surface representation. A reviews of the RBF theory is given by Powell [30] and Buhmann [1]. Some examples of RBFs are, thin plate splines $\phi(r) = r^2 \log(r)$, Hardy's multiquadric $\emptyset(r) = \sqrt{r + \alpha^2}$ and Gaussian RBF given by:

$$\phi(r) = e^{-(\alpha r)^2} \tag{2.1}$$

The Gaussion RBF given in equation (2.1) is adapted in this paper for modelling leaf surface. The RBF precision depends on the parameterα [12, 7]. Majdisova [22, 23] introduced an interpolat RBF approach and they clarified the shaped parameter experimentally. Rippa [34] proposed an optimal value of the RBF parameter based on minimizes the cost function [32]. The approximation of the RBF parameter discussed by many authors and can be found in [2, 3, 4, 9, 14, 18, 22, 23, 33, 37, 38, 39, 40].

Now, the interpolation RBF $\Gamma : D \subset \Re^2 \to \Re$to the function$z(\bar{x})$is given by

$$\Gamma(x) = \sum_{i=1}^{N} \lambda_i \psi(\|x - x_i\|) = \sum_{i=1}^{N} \lambda_i \psi(r_i), \quad \bar{x} \in \mathrm{R}^2 \tag{2.2}$$

where

$$\Lambda\lambda = z \quad with \quad \Lambda_{ij} = \psi(\|x_j - x_i\|) \quad i, j = 1, \ldots, N \tag{2.3}$$

And $\psi_{i,j}(r)$ is the RB, $r_i = \|x_j - x_i\|_2$is the distance between the interpolation point $\bar{x}_j$and the rest of the data points $\bar{x}_i$, $z_j(\bar{x}_j) = \Gamma(\bar{x}_j), j = 1, ..., N$, $N$is the number of nodes and λ are the interpolation coefficients.

The radial basis function is suitable for scattered data in d-dimensional and can be extended by polynomial $P_k(\mathbf{x})$ of degree (k) to improve the stability of the method. In this research we investigated two cases, in the first case we added to the Gaussion RBF given in equation (2.1) a linear polynomial of degree one while in the second case we added to the Gaussion RBF a third order polynomial. In the following paragraph we will explain the second case. The third order polynomial is given by:

$$\mathrm{P}_3(\bar{\mathrm{x}}) = \mathrm{b}_0 + \mathrm{b}_1 \mathrm{x} + \mathrm{b}_2 \mathrm{y} + \mathrm{b}_3 x^2 + \mathrm{b}_4 xy + \mathrm{b}_5 y^2 + \mathrm{b}_6 x^3 + \mathrm{b}_7 x^3 y + \mathrm{b}_8 xy^3 + \mathrm{b}_9 y^3 = \bar{\mathrm{b}}^{\mathrm{T}} \bar{\mathrm{x}}$$

where $\bar{x} = [1, x, y, x^2, xy, y^2, x^3, x^3 y, xy^3, y^3]^T$. So equation (2.2) becomes:

$$\Gamma(\bar{x}_j) = \sum_{i=1}^{N} \lambda_i \psi_{i,j}(r) + \sum_{k=1}^{3} \mathrm{b_k} P_k(\bar{x}_j) = \sum_{i=1}^{N} \lambda_i \psi_{i,j} + \bar{\mathrm{b}}^{\mathrm{T}} \bar{x}, j = 1, \ldots, N \tag{2.4}$$

where $P_k(\bar{x}_j)$ is a polynomial, γ_i and b_k are the interpolation coefficients and $\sum_{j=1}^{N} \lambda_i P_k(\bar{x}_j) = \bar{0}$. Now, rewrite equation (2.4) in a matrix form as follows:

$$\Lambda \bar{\lambda} = z \tag{2.5}$$

where

$$\Lambda = \begin{bmatrix} \boldsymbol{\Upsilon} & \boldsymbol{\aleph} \\ \boldsymbol{\aleph}^{\boldsymbol{T}} & \mathbf{0} \end{bmatrix}, \boldsymbol{\Upsilon} = \begin{bmatrix} \psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,N} \\ \psi_{2,1} & \psi_{2,2} & \cdots & \psi_{2,N} \\ \vdots & \vdots & \vdots & \vdots \\ \psi_{N,1} & \psi_{N,2} & \cdots & \psi_{N,N} \end{bmatrix},$$

$$\boldsymbol{\aleph} = \begin{bmatrix} 1 & \mathrm{x}_1 & \mathrm{y}_1 & \mathrm{x}_1^2 & \mathrm{x}_1 y_1 & y_1^2 & x_1^3 & x_1^3 y_1 & x_1 y_1^3 & y_1^3 \\ 1 & \mathrm{x}_2 & \mathrm{y}_2 & \mathrm{x}_2^2 & \mathrm{x}_2 y_2 & y_2^2 & x_2^3 & x_2^3 y_2 & x_2 y_2^3 & y_2^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_N & y_N & x_N^2 & x_N y_N & y_N^2 & x_N^3 & x_N^3 y_N & x_N y_N^3 & y_N^3 \end{bmatrix},$$

$$\bar{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \\ b_0 \\ b_1 \\ \vdots \\ b_N \end{bmatrix} \quad \text{and} \quad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

The truncated singular value decomposition (TSVD) is applied in this paper to solve the linear system given in equation 2.5. however, The smallest SV's are ignored if they were less than or equal to the product of the ε(machine epsilon) with the largest SV [21].

2.1. **Numerical Experiment for a scattered data set.**

The Gaussion RBF enhancing a linear (GRBFL) and cubic (GRBFC) polynomial is examined in this section locally and globally. A numerical investigation for GRBFC and GRBFL is performed on a data points (consists of 100 point and 33 point) and five test fuctions taken from Franke [7], see figure 1. The root mean square error (RMS) given in equation (2.6) is used to measure the accuracy of both methods.

$$\mathrm{RMS} = \sqrt{\frac{\sum_{k=1}^{m} [\Gamma(x_k) - z_k]^2}{m}} \tag{2.6}$$

where $\Gamma(x_k), k = 1, 2, \ldots, m$are the interpolation GRBFC and $z_i$are the given function values.

2.1.1. *Results and Discussion.*

We used the 100 points ($N = 100$) to creat one global Gaussion RBF (table 3), global Gaussion RBF with linear polynomial (global GRBFL) Table 1, and global Gaussion RBF with cubic polynomial (global GRBFC) see equation (2.4), table 3. Furthermore, we used the global GRBFC, GRBFL and GRBF to estimate the values of the 33 points, see figure 1. howevere, the local Gaussion RBF with cubic or linear polynomial (local GRBFC or local GRBFL) is construct locally from the nearest 20 points for each point of the 33 points ($n = 20 \subset N = 100$), then we used the local GRBFC and local GRBFL to approximate the value for that specific

TABLE 1. A comparison of the RMS error using the global and the local Gaussion RBF with linear polynomial.

Function	**C**	**with Linear polynomial**	
		Global Gaussion RBF	**Local Gaussion RBF**
F1	6.2684	4.2 e-003	7.9 e-003
F2	4.0120	1.1 e-003	1.2 e-003
F3	3.9828	4.3 e-005	9.2 e-004
F4	4.0434	2.7 e-004	2.7 e-003
F5	4.1169	3.8 e-004	6.8 e-004

TABLE 2. A comparison of the RMS error using the global and the local gaussion RBF with cubic polynomial.

Function	**C**	**with cubic polynomial**	
		Global Gaussion RBF	**Local Gaussion RBF**
F1	6.9210	7.6e-003	9.7 e-003
F2	4.7462	5.7 e-004	8.9 e-004
F3	2.3684	9.7 e-008	5.0 e-005
F4	4.3631	7.3 e-005	2.5 e-003
F5	4.0212	7.6 e-005	3.4 e-004

TABLE 3. A comparison of the RMS error using the global and the local gaussion RBF.

Function	**C**	**Without polynomial**	
		Global Gaussion RBF	**Local Gaussion RBF**
F1	5.1410	4.4e-003	5.0 e-003
F2	3.3918	1.2e-004	5.9e-004
F3	2.3528	6.9e-006	2.3e-005
F4	4.3286	1.2e-005	9.1e-004
F5	1.6309	7.9e-005	1.3e-004

point. In both cases we used Rippa algorithm [34] to estimate the GRBF shape parameter c globally using the $N = 100$points.

Our numerical investigation given in table 1 represent that the RMS error for five fuctions using either global GRBFL method or local GRBFL method. We observed from Table 1 the follwoing. Firstly, the global GRBFL produces more accurately RMS error than using the local GRBFL for the five functions. Secondly,

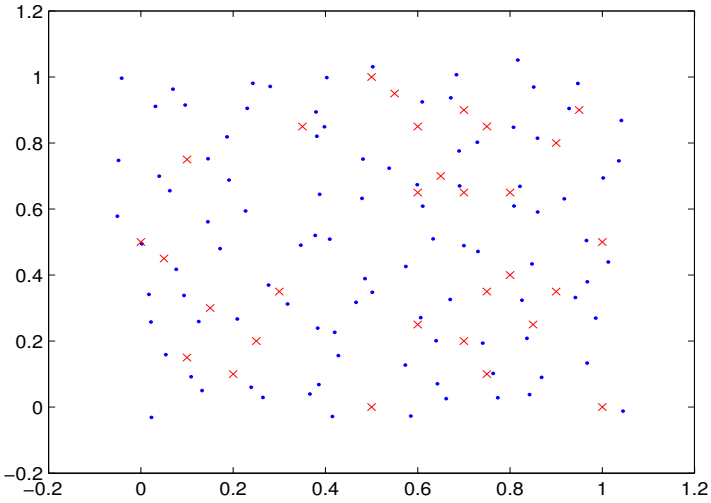


FIGURE 1. The Franke data sets. The blue dots are 100 point and the red dots are 33 point defined on the unit square $[0,1]^2$.

the global GRBFL requires less computational than using the local GRBFL because a dense matrix of size 23×23 (see equation (5)) demands to be solved at each time the local method is used. Finally, the parameter (c) is different for each function (alternate between 3 and 6.5) and depends on the collected data that used to build the GRBFL. Consequently, this is confirms that the parameter has large impact on the RBF approximation. Table 2 shows the RMS error for five fuctions using either global GRBFC or local GRBFC method. The tendency depicted in Table 2 seems consistent with the given observations in Table 1. Furthermore, the GRBFC is better than the GRBFL.

One observation from table 3 that the RMS obtained by global GRBF is more precise than the one obtained by local GRBF method. Additionally, for the functions F1, F2 and F4 the global GRBF was more accurate than the global GRBFC but it's the converse for the functions F3 and F5. In conclusion, the global GRBF (table 3) and the global GRBFC (table 2) produces the best RMS error of all methods, Therefore we carry out this result and we apply it to a real Anthurium leaf data in the following section.

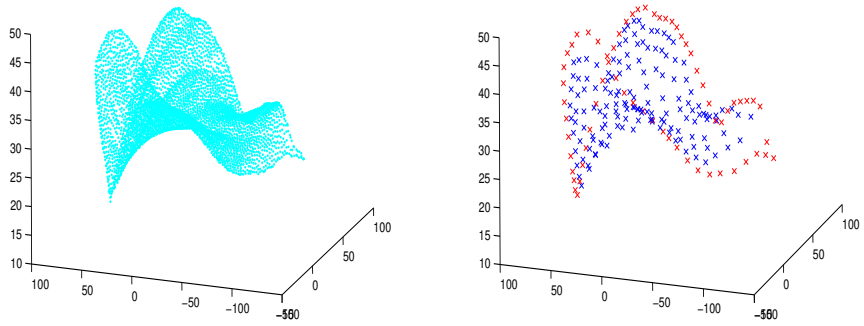
3. Application of the GRBFC technique to a real leaf data set

The proposed Gaussion Radial basis function with cubic polynomial (GRBFC) technique is verified here by applying it to estimate the surface values for Anthurium given in figure 2. The leaf data collected using a laser scanner device by Loch [17, 20] while the leaf boundary points were collected using a software called PointPicker by Hanan [10, 11]. The sampled points for the Anthurium leaf were 4,688 points respectively and the boundary points were 79 points respectively, see figure 2.

The GRBF and GRBFC method can not be apply directly to model the leaf surface, therfore, two steps are essential. Firstly, the choice of the shape parameter

for the Gaussion RBF. Secondlly, the reference plane for the scanned leaf points may not essentially match with the xy-plane, see figure 3. Additionally, there are possibility of the multi valued surfaces of the leaf data, see oqielat [24, 27]. So a new reference plane for the leaf data is required.

In this paper the orthogonal distance regression plane is used as reference plane for the leaf data. This plane is based on minimizing the perpendicular distances from the leaf surface points to the plane. Furthermore, the coordinate system is rotated via a rotation matrix that rotates the unit normal vector of the orthogonal distance regression plane about the x-axis and y-axis, to obtain thexy-plane as the new plane, see figure 3.



(a) (b)

FIGURE 2. (a) The 4,688 scanned points for the Anthurium leaf in 3D given on the left side; (b) The 212 interior points. The red cross represent the 79 boundary points of the Anthurium leaf.

The orthogonal distance regression plane given in equation (3.1) is applied to fit the given leaf data $(x_i, y_i) = z_i$.

$$g(x, y) = ax + by + d. \tag{3.1}$$

Minimizing $g(a, b, d) = \sum_{i=1}^{N} \frac{|ax_i+by_i+d|^2}{a^2+b^2}$ to obtain the best fit by evaluating the partial derivative with respect to (d) to be zero implies that knowing the values of a, b, c *and* d, so equation (3.1) becomes

$$g(a, b) = \sum_{i=1}^{N} \frac{|a(x_i - x_0) + b(y_i - y_0)|^2}{a^2 + b^2}. \tag{3.2}$$

where (x_0, y_0) is the data points center. The matrix representation of equation (3.2) implies that

$$g(v) = \frac{(v^T M^T)(Mv)}{(v^T v)} = \frac{v^T (M^T M) v}{(v^T v)} = \frac{v^T A v}{v^T v}$$

where $v^T = [a, b]$, M is a two columns matrix contains in the first column the elements $(x_i - x_0)$ and in the second column $(y_i - y_0)$, $i = 1, 2, \ldots N$. Notice that

the normal vector of the orthogonal distance regression plane is the singular vector of the matrix M.

Now and after the leaf data is projected into the orthogonal distance regression plane. The Gaussion RBF with cubic polynomial method is applied to reconstruct the surface values for the Anthurium and Frangipani leaf shown in figure 4.

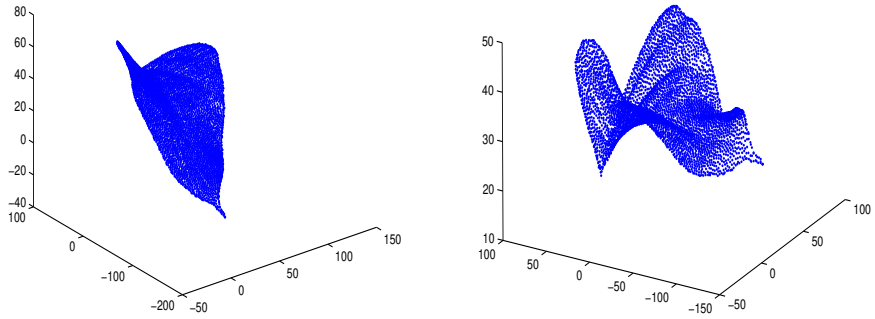
3.1. **Numerical Experiments for the Leaf Surface.**

In this section the results of applying the GRBF and GRBFC method into a real Anthurium leaf data is given. The Anthurium leaf data consist of 4,767 points, see figure 2. Two measuring errors are used to evaluate our method based on the root mean square error (RMS) and the relative maximum error given in equations (2.6) and (3.3) respecxtively.

$$\text{Maximum Error} = \frac{\max\left(|\Gamma(a_i, b_i) - z_i|\right)}{\max(z_i) - \min(z_i)}, \tag{3.3}$$

where $\Gamma(a_i, b_i), i = 1, 2, \ldots, m$are the interpolant values at the data points (m) and $z_i$are the given function values at the same data points.

One Global Gaussion RBF with cubic polynomial is build from 212 points to model the Anthurium leaf, see figure 2. The rest of the Anthurium leaf points after removing the 212 points are 4,555 points, is then used to measure the accuracy of the GRBF and GRBFC method. The results of employing the GRBF method and the



(a) (b)

FIGURE 3. (a)The Anthurium leaf data before projection into a new reference plane and (b) after projection.

proposed GRBFC globally to model the surface of Anthurium leaf is given in tables 4. The RMS and the maximum error are calculated for 4555 Anthurium points to evaluate our method. The Gaussion RBF shape parameter is computed randomily and found that the parameter values of 0.01 gave the best result for the GRBF and GRBFC to model Anthurium leaf. Notice that from Table 4 the relative maximum error generated using GRBF and GRBFC is less accurate than the relative RMS. Furthermore, using the GRBF produced slightly more accurate representation of the leaf surface than using GRBFC. Thus, adding a cubic polynomial to the GRBF

TABLE 4. The relative RMS and relative maximum error for the Anthurium leaf data points computed using the global GRBF and global GRBFC method.

	Global GRBF	**Global GRBF With Cubic Polynomial**
Maximum error	5.9e-002	9.3e00-2
Relative RMS	1.0e-002	1.9e00-2
Number of point tested	4555	4555
Number of Boundary points	79	79
The RBF shape Parameter (c)	0.01	0.01

did not improve the accuracy of the method since GRBF is strictly positive definite and its approximation converges exponentially. In conclusion, the proposed GRBFC technique produced an accuarte leaf surface representation (see figure 4) but it did not improve the accuracy of the GRBF method.

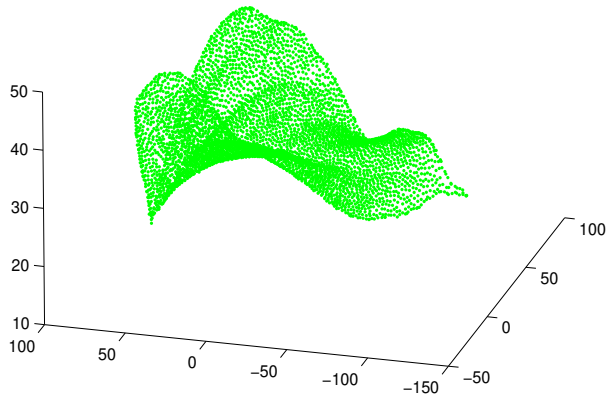


FIGURE 4. The Anthurium leaf surface model constructed from the points (shown in figure 2) using the global GRBFC method.

4. CONCLUSIONS

The research given in this article described two interpolation method including the GRBF and the GRBFC to reconstrct the leaf surface from three-dimensional data. The methods are validated using two sets of data points where the first set is taken from franke [7] while the second set is a real data for anthurium leaf. The Gaussion RBF with cubic polynomial technique shown to give an accurate representation of

the leaf (see figure 4). Furthermore, this model has many application and can be employed to model a droplet of liquid movement on a leaf surface.

References

[1] M. D, Buhmann, *Radial Basis Functions: Theory and Implementations.*, Cambridge university press textbf12 (2003).

[2] R. E. Carlson and T. A. Foley, *The parameter R2 in multiquadric interpolation*, Comput. Math. Appl textbf21 (1991) 29-42.

[3] J. C. Carr, R. K. Beatson, J. B. Cherrie, T. J. Mitchell, W. R. Fright, B. C. McCallum, and T. J. Evans, *Reconstruction and representation of 3D objects with radial basis functions,* ACM SIGGRAPH, (2001) 67-76.

[4] J. C. Carr, R. K. Beatson, B. C. McCallum, W. R. Fright, T. J. McLennan, and T. J. Mitchell, *Smooth surface reconstruction from noisy range data,* ACM Graphite (2003) 119-126.

[5] O. Davydov and F. Zeilfelder, *Scattered data fitting by direct extension of local polynomials to bivariate splines,* Advances in Computational Mathematics **21(3-4)** (2004) 223-271.

[6] Dorr GJ, Kempthorne D, Mayo LC, ForsterWA, Zabkiewicz JA, McCue SW, Belward JA, Turner IW, Hanan J, *Towards a model of spray–canopy interactions: interception, shatter, bounce and retention of droplets on horizontal leaves,* Ecol.Model **290** (2014) 94-101. (doi:10.1016/j. ecolmodel.2013.11.002)

[7] R. Franke, textitScattered data interpolation: Tests of some methods, Mathematics of Computation **38 157** (1982).

[8] Y.T. Gu, Q.X.Wang, K.Y.Lam and K.Y. Dai, *A pseudo-elastic local meshless method for analysis of material nonlinear problems in solids,* Engineering Analysis with Boundary Elements **31 9** (2007) 771-782.

[9] E. Gregory,Fasshauer, G. E., Zhang, J. G., *On choosing "optimal" shape parameters for RBF approximation,* Numerical Algorithms **45 (1-4)** (2007) 345-368.

[10] J. Hanan, B. Loch, T. McAleer, *Processing Laser Scanner Plant Data To Extract Structural Information. Advanced Computational Modelling Centre,* University of Queensland, Australia (2007).

[11] J. Hanan, M. Renton, E. Yorston, *Simulating and visualising spray deposition on plant canopies. Advanced Computational Modelling Centre,* University of Queensland, Australia (2003).

[12] R. L. Hardy, *Theory and applications of the multiquadric-biharmonic method,* Comput. Math. Appl, **19** (1990) 163-208.

[13] R. L. Hardy, *Multiquadratic Equations of Topography and Other Irregular Surfaces,* Journal of Geophysical Research **76** (1971) 1905-1915.

[14] Hon, Y.-C., Sarler, B., Yun, D.-F., *Local radial basis function collocationmethod for solving thermo-driven fluid-flow problems with free surface,* Engineering Analysis with Boundary Elements **57** (2015) 2-8.

[15] D.M Kempthorne, Turner IW, Belward JA, McCue SW, Barry M, Young J, Dorr GJ, Hanan J, Zabkiewicz JA, *Surface reconstruction of wheat leaf morphology from three-dimensional scanned data,* Funct. Plant Biol. **42** (2015) 444-451. (doi:10.1071/ FP14058).

[16] D.M Kempthorne, Turner IW, Belward JA. *A comparison of techniques for the reconstruction of leaf surfaces from scanned data* SIAM J. Sci (2015).

[17] B. Loch, J. Belward, J. Hanan, *Application of Surface Fitting Techniques for the Representation of Leaf Surfaces,* Advanced Computational Modelling Centre, University of Queensland, Australia (2003).

[18] M. Li, Chen, W., Chen, C., *The localized RBFs collocation methods for solving high dimensional PDEs,* Engineering Analysis with Boundary Elements **37 10** (2013) 1300-1304.

[19] C. M Lisa, S.. McCue1, T. Moroney, W. Alison, D. Kempthorne, J. Belward and I. Turner, *Simulating droplet motion on virtual leaf surfaces,* Royal society open science **10** (2016).

[20] B. Loch, *Surface Fitting for the Modelling of Plant Leaves,* Ph.D. Thesis, University of Queensland, (2004).

[21] T. Moroney, *An investigation of a finite volume method incorporating radial basis functions for simulating nonlinear transport,* Ph.D. Thesis, Australia, http://www.sci.qut.edu.au/about/staff/mathsci/computational/moroneyt.jsp, (2006).

[22] Z.Majdisova, V.Skala, *Big geo data surface approximation using radial basis functions: A comparative study,* Computers and Geosciences, **109** (2017) 51-58.
[23] Z.Majdisova, V.Skala, *Radial basis function approximations: comparison and applications,* Applied Mathematical Modelling, textbf51 (2017) 728-743.
[24] M.N.Oqielat,J.A. Belward, and I.W. Turner, *A Hybrid Clough-Tocher method for surface fitting with application to leaf data,* Applied Mathematical Modelling **33** (2009) 2582-2595.
[25] M.N. Oqielat, J.A. Belward, I.W. Turner, B.I. Loch, *A hybrid Clough–Tocher radial basis function method for modelling leaf surfaces,* International Congress on Modelling and Simulation, Modelling and Simulation Society of Australia and New Zealand (2007) 400-406.
[26] M. N. Oqielat, Turner IW, Belward JA, McCue SW., *Modelling water droplet movements on a leaf surface,* Math. Comput. Simulat **81** (2011)1553-1571. doi:10.1016/J.matcom.2010.09.003.
[27] M.N. Oqielat, *Surface Fitting Methods for Modelling Leaf surface from scanned data,* Journal of King Saud University (2017).
[28] M.Oqielat, O. Ogilat,N. Al-Oushoush and A. Sami B., *Radial Basis Function Method For Modelling Leaf Surface from Real Leaf Data,* Australian Journal of Basic and Applied Sciences textbf11 13)2017) 103-111.
[29] M. Oqielat, *Comparison of surface fitting methods for modelling leaf surfaces,* Italian Journal Of Pure and Applied Mathematics (2017).
[30] M. Powell, *The theory of radial basis function approximation in 1990,* Advances in Numerical Analysis, Wavelets, Subdivision Algorithms and Radial Functions (1991).
[31] P. Prusinkiewicz, *Modelling of spatial structure and development of plants: a review,* Scientia Horticulturae **74** (1998) 113-149.
[32] W.H. Press, B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, *Numerical Recipes in C. The Art of Scientific Computing,* Cambridge University Press, Cambridge, (1992).
[33] D.W. Pepper, Rasmussen, C., Fyda, D., *A meshless method using global radial basis functions for creating 3-d wind fields from sparse meteorological data,* Computer Assisted Methods in Engineering and Science **21 3-4** (2014) 233-243.
[34] S. Rippa,*An algorithm for selecting a good value for the parameter c in radial basis function interpolation,* Advances in Computational Mathematics **11** (1999) 193-210.
[35] P. Room, J. Hanan, and P. Prunsinkiewicz, *Virtual plants: new perspectives for ecologists, pathologists and agricultural scientist,* Trends in Plant Science **1 1** (1996) 33-38.
[36] H. Sinoquet, S. Thanisawanyangkura, H. Mabrouk, and P. Kasemsap, *Characterization of the light environment in canopies using 3D digitising and image processing,* Annals of Botany **82** (1998) 203-212.
[37] M. Scheuerer, *An alternative procedure for selecting a good value for the parameter c in RBF-interpolation,* AdvComput Math. **34 1** (2011) 105-126.
[38] V. Skala, *RBF Interpolation with CSRBF of Large Data Sets,* International conference on Computational Science (2017) 12-14.
[39] V. Skala, *A practical use of radial basis functions interpolation and approximation,* Revista Investgation Operational **37 2** (2016) 137-145.
[40] V. Skala, *Fast interpolation and approximation of scattered multidimensional and dynamic data using radial basis functions,* WSEAS Transactions on Mathematics **12** (2013).
[41] M. Oqielat, *Scattered Data Approximation Using Radial Basis function with Cubic polynomial Reproductionfor modelling Leaf Surface,* Computational mathematics and modelling, (2018).
[42] M. Oqielat; O. Ogilat, *Approximation of Radial Basis function with linear polynomial: Comparison and Application,* Accepted (2018).

Moa'ath N. Oqielat, Department of Mathematics, Faculty of Science, Al-Balqa' Applied University, Salt 19117, Jordan.
E-mail address: moaathoqily@bau.edu.jo

Osama Ogilat, Department of Mathematics, Faculty of Arts & Science,, Al-Ahliyya Amman University, Box.19328. Amman. Jordan.
E-mail address: o.oqilat@ammanu.edu.jo