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FUZZY PARAMETERIZED FUZZY SOFT METRIC SPACES

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Abstract. Starting with the knowledge of fuzzy parameterized fuzzy soft point in $\mathfrak{fpfs}$-sets, we inaugurate the notion of $\mathfrak{fpfs}$-metric space. We investigate some topological structures of $\mathfrak{fpfs}$-metric space including $\mathfrak{fpfs}$-open ball, $\mathfrak{fpfs}$-closed ball, $\mathfrak{fpfs}$-diameter and $\mathfrak{fpfs}$-neighborhood of $\mathfrak{fpfs}$-set. We inaugurate some motivating results based on $\mathfrak{fpfs}$-metric space. To handle the decision-making problem we construct an algorithm and present a novel application for $\mathfrak{fpfs}$-metric space.

1. Introduction

A metric space is a loop in mathematics, for which spaces between all members of the set are defined. "A metric on a space stimulates topological attributes like open and closed sets, which contribute to the survey of more abstract topological spaces. One major motivation for studying them are to better understand the spaces of functions. In 1906, metric spaces were introduced by Frechet [10] in his PhD dissertation on functional analysis. In the modern view, the concept of a metric space is just an axiomatization of the notion of distance. It is among the more straightforward axiomatizations, especially to modern students who see axiomatic systems early on".

In 1965, Zadeh [28] created the idea of fuzzy set as a abstraction of crisp set. Fuzzy set theory has many applications in several areas including social sciences, physics, engineering, economics, computer science and medical sciences. In 1999, Molodtsov [11] devised the notion of soft set as a mathematical instrument to deal with the troubles having fuzziness. Beaula and Gunaseeli [4] introduced some properties including fuzzy soft open and fuzzy soft closed balls on fuzzy soft metric spaces. Çağman *et al.* [5, 6] presented fuzzy soft set theory and $\mathfrak{fpfs}$-set theory with applications. Das and Samanta [7] established soft metric space and investigated some of its properties with Cantor's intersection theorem. Maji *et al.* [12] established a new algorithm for soft set in decision-making problem. They investigated the notion of the operator ωCl_θ and study some properties of it with ω-regularity. "Riaz *et al.* [14, 15, 16, 17, 18, 19, 20] inaugurated some concepts of soft sets together with soft algebra and measurable soft mappings. They established certain properties of

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soft metric spaces and introduced Baire's category theorem. They studied fuzzy parameterized fuzzy soft set($\mathfrak{fpfs}$-set), $\mathfrak{fpfs}$-topology and $\mathfrak{fpfs}$- compact spaces with some substantial proposals and established certain applications of $\mathfrak{fpfs}$-set to the decision-making problems. They introduced some operations on fuzzy neutrosophic soft set ($\mathfrak{fns}$-set) by utilizing the theories of fuzzy sets, soft sets and neutrosophic sets. They introduced $\mathfrak{fns}$-mappings by using cartesian product with relations on $\mathfrak{fns}$-sets and establish some results on fixed points of $\mathfrak{fns}$-mapping". Subramanian *et al.* [26] studied the ideal of χ^2 over fuzzy p-metric spaces defined by musielak. Zorlutuna and Atmaca [27] presented $\mathfrak{fpfs}$-topology with some important results and $\mathfrak{fpfs}$-mappings. Soft set theory, $\mathfrak{fs}$-set theory and $\mathfrak{ifs}$-set theory has examined by many explorers in the last decade (See [1, 2, 3, 8, 9, 11, 13, 21, 22, 23, 24, 25]). The aim of our paper is to discuss various properties of $\mathfrak{fpfs}$-set theory. We demonstrate $\mathfrak{fpfs}$-metric space and innovate several ideas including $\mathfrak{fpfs}$-diameter, $\mathfrak{fpfs}$-open ball, $\mathfrak{fpfs}$-closed ball and $\mathfrak{fpfs}$-neighborhood. We establish a new algorithm for $\mathfrak{fpfs}$-metric space to the decision-making problem. This paper can form the conjectural basis for auxiliary applications of $\mathfrak{fpfs}$-metric space.

2. Preliminaries

Definition 2.1. [5, 27] *"Let X be the universal set and $\widetilde{P}(X)$ is the power set of all fuzzy subsets of X, R be the set of attributes and $A \subseteq R$. A fuzzy parameterized fuzzy soft set ($\mathfrak{fpfs}$-set) is a mathematical function $\gamma : R \rightarrow \widetilde{P}(X)$ such that $\gamma_A(\zeta) = \phi$ if $\mu_A(\zeta) = 0$ for $\zeta \in R$. The $\mathfrak{fpfs}$-set is denoted by*

$$F_A = \{(\mu_A(\zeta)/\zeta, \gamma_A(\zeta)) : \zeta \in R, \gamma_A(\zeta) \in \widetilde{P}(X); \mu_A(\zeta), \gamma_A(\vartheta) \in [0,1], \vartheta \in X\}.$$

The value $\gamma_A(\zeta)$ is a fuzzy set known as ζ-element of $\mathfrak{fpfs}$-set F_A $\forall$ $\zeta \in R$.

Definition 2.2. [5, 27] *Let F_A be a $\mathfrak{fpfs}$-set over X. If $\lambda_A(\zeta) = \phi$ $\forall$ $\zeta \in R$ then F_A is known as A-empty $\mathfrak{fpfs}$-set. It is represented as F_{ϕ_A}.*
If $A = \phi$, then A-empty $\mathfrak{fpfs}$-set is named as an empty $\mathfrak{fpfs}$-set symbolized by F_ϕ.

Definition 2.3. [5, 27] *Let F_A be a $\mathfrak{fpfs}$-set over X. If $\gamma_A(\zeta) = X$ and $\mu_A(\zeta) = 1$ $\forall$ $\zeta \in R$ then F_A is known as A-universal $\mathfrak{fpfs}$-set. It is represented as $F_{\widetilde{A}}$.*
If $A = R$, then A-universal $\mathfrak{fpfs}$-set is said to be universal or absolute $\mathfrak{fpfs}$-set written as $F_{\widetilde{R}}$".

Definition 2.4. [17, 27] *A $\mathfrak{fpfs}$-set F_A is said to be a $\mathfrak{fpfs}$-point, denoted by $\zeta(F_A)$, if $A \subseteq R$ is fuzzy singleton and $(\mu(\zeta)/\zeta) \in A$; $F(\mu(\zeta)/\zeta) = \gamma^{\zeta}_{F_A}(\vartheta)$ where $\gamma^{\zeta}_{F_A}(\vartheta) \neq \widetilde{\phi}$ and $F(\mu(\zeta')/\zeta') = \widetilde{\phi}$ $\forall$ $(\mu(\zeta')/\zeta') \in R - \{(\mu(\zeta)/\zeta)\}$.*

Proposition 2.5. *"The $\mathfrak{fpfs}$-union of any collection of $\mathfrak{fpfs}$-point can be considered as a $\mathfrak{fpfs}$-set and every $\mathfrak{fpfs}$-set can be expressed as the $\mathfrak{fpfs}$-union of all $\mathfrak{fpfs}$-points".*

$$F_A = [\widetilde{\bigcup_{\zeta(F_B)\widetilde{\in} F_A}} \zeta(F_B)].$$

Proposition 2.6. *Let F_A and F_B be two $\mathfrak{fpfs}$-sets then $F_A \widetilde{\subseteq} F_B$ if and only if $\zeta(F_C)\widetilde{\in} F_A$ implies $\zeta(F_C)\widetilde{\in} F_B$ and hence $F_A = F_B$ if and only if $\zeta(F_C)\widetilde{\in} F_A$ if and only if $\zeta(F_C)\widetilde{\in} F_B$.*

3. $\mathfrak{fpfs}$-Metric Space

Definition 3.1. *Let X be a non-empty fuzzy set and R be the fuzzy set of attributes then a mapping $\varepsilon : R \to X$ is called an $\mathfrak{fpfs}$-element of X. An $\mathfrak{fpfs}$-element ε is said to be in $\mathfrak{fpfs}$-set F_A of X, which is symbolized by $\varepsilon \widetilde{\in} F_A$ if*
$\mu_\varepsilon(\zeta) \widetilde{\leq} \mu_{F_A}(\zeta)$ *and* $\gamma^\zeta_\varepsilon(\vartheta) \widetilde{\leq} \gamma^\zeta_{F_A}(\vartheta)\ \forall\ \zeta \in R, \vartheta \in X$.
Therefore for an $\mathfrak{fpfs}$-set F_A of X w.r.t the index set $\mathbb{R}$,
we have $F_A(e) = \{\varepsilon(\zeta), \varepsilon \widetilde{\in} F_A\},\ \zeta \in R$.

Definition 3.2. *"Let $\mathbb{R}$ be the set of real numbers and $\mathfrak{B}(\mathbb{R})$ be the collection of all non-empty bounded fuzzy subsets of $\mathbb{R}$ and E is the set of attributes, A be a fuzzy subset of E. Then the mathematical function $F : A \to \mathfrak{B}(\mathbb{R})$ is known as $\mathfrak{fpfs}$-real set. It is symbolized by F_A. If F_A is a singleton $\mathfrak{fpfs}$-real set then F_A with the corresponding $\mathfrak{fpfs}$-element is said to be $\mathfrak{fpfs}$-real number. The $\mathfrak{fpfs}$-real numbers is symbolized by $\widetilde{r}, \widetilde{s}, \widetilde{t}$ whereas $\overline{r}, \overline{s}, \overline{t}$ represent specific type of $\mathfrak{fpfs}$-real numbers such that $\overline{r}(\mu(\zeta_i)/\zeta_i) = r_{\gamma(r)}\ \forall\ \zeta_i \in A$".*

Remark. *Let $\mathfrak{P}$ be a assembling of $\mathfrak{fpfs}$-points then the $\mathfrak{fpfs}$-set generated by $\mathfrak{P}$ can be represented by $\mathfrak{fpfs}G(\mathfrak{P})$ and the assembling of all $\mathfrak{fpfs}$-points of a $\mathfrak{fpfs}$-set F_A be denoted by $\mathfrak{fpfs}C(\mathfrak{P})$.*

Definition 3.3. *Let $A \subseteq R$ and let $F_{\widetilde{R}}$ be an absolute $\mathfrak{fpfs}$-set. Let $(A)^*$ represents the set of all non-negative $\mathfrak{fpfs}$-real numbers. The $\mathfrak{fpfs}$-metric using $\mathfrak{fpfs}$-points is defined as follows:*
A mapping $d : \mathfrak{fpfs}C(F_{\widetilde{R}}) \times \mathfrak{fpfs}C(F_{\widetilde{R}}) \to \mathbb{R}(A)^*$ is called a $\mathfrak{fpfs}$-metric if d gratifies the following conditions,
$(\mathfrak{fpfs}M_1)$ $d(\zeta(F_{A_1}), \zeta'(F_{A_2})) \widetilde{\geq} \overline{0}\ \forall\ \zeta(F_{A_1}), \zeta'(F_{A_2}) \widetilde{\in} \mathfrak{fpfs}C(F_{\widetilde{R}})$.
$(\mathfrak{fpfs}M_2)$ $d(\zeta(F_{A_1}), \zeta'(F_{A_2})) = \overline{0}$ *if and only if* $\zeta(F_{A_1}) = \zeta'(F_{A_2})$.
$(\mathfrak{fpfs}M_3)$ $d(\zeta(F_{A_1}), \zeta'(F_{A_2})) = d(\zeta'(F_{A_2}), \zeta(F_{A_1}))\ \forall\ \zeta(F_{A_1}), \zeta'(F_{A_2}) \widetilde{\in} \mathfrak{fpfs}C(F_{\widetilde{R}})$.
$(\mathfrak{fpfs}M_4)$ $d(\zeta(F_{A_1}), \zeta''(F_{A_3})) \widetilde{\leq} d(\zeta(F_{A_1}), \zeta'(F_{A_2})) + d(\zeta'(F_{A_2}), \zeta''(F_{A_3}))$
$\forall\ \zeta(F_{A_1}), \zeta'(F_{A_2}), \zeta''(F_{A_3}) \widetilde{\in} \mathfrak{fpfs}C(F_{\widetilde{R}})$.
The $\mathfrak{fpfs}$-set $F_{\widetilde{R}}$ with the $\mathfrak{fpfs}$-metric d is called $\mathfrak{fpfs}$-metric space and is denoted by $(F_{\widetilde{R}}, d)$. The conditions $(\mathfrak{fpfs}M_1), (\mathfrak{fpfs}M_2), (\mathfrak{fpfs}M_3)$ and $(\mathfrak{fpfs}M_4)$ are said to be $\mathfrak{fpfs}$-metric axioms.

Example 3.4. *Let X be a non-empty set and $A \subseteq R$ be the set of attributes. Let $F_{\widetilde{R}}$ be the universal $\mathfrak{fpfs}$-set. We define $d : \mathfrak{fpfs}C(F_{\widetilde{R}}) \times \mathfrak{fpfs}C(F_{\widetilde{R}}) \to \mathbb{R}(A)^*$ by,*
$d(\zeta(F_{A_1}), \zeta'(F_{A_2})) = \overline{0}$ *if* $\zeta(F_{A_1}) = \zeta'(F_{A_2})$ *and*
$d(\zeta(F_{A_1}), \zeta'(F_{A_2})) = \overline{1}$ *if* $\zeta(F_{A_1}) \neq \zeta'(F_{A_2})\ \forall\ \zeta(F_{A_1}), \zeta'(F_{A_2}) \widetilde{\in} F_{\widetilde{R}}$.
Here d satisfies all the $\mathfrak{fpfs}$-metric axioms. So, d is a $\mathfrak{fpfs}$-metric on the $\mathfrak{fpfs}$-set $F_{\widetilde{R}}$. d is called discrete $\mathfrak{fpfs}$-metric space on the $\mathfrak{fpfs}$-set $F_{\widetilde{R}}$ and $(F_{\widetilde{R}}, d)$ is said to be discrete $\mathfrak{fpfs}$-metric space.

Definition 3.5. *A mapping $d : \mathfrak{fpfs}C(F_{\widetilde{R}}) \times \mathfrak{fpfs}C(F_{\widetilde{R}}) \to \mathbb{R}(A)^*$ is called a $\mathfrak{fpfs}$-pseudo metric on the $\mathfrak{fpfs}$-set $F_{\widetilde{R}}$ if d gratifies the following conditions:*
$(\mathfrak{fpfs}PM_1)$ $d(\zeta(F_{A_1}), \zeta'(F_{A_2})) = \overline{0}$ *if* $\zeta(F_{A_1}) = \zeta'(F_{A_2})$ *or* $\zeta'(F_{A_2}) \widetilde{\subset} \zeta(F_{A_1})$.
$(\mathfrak{fpfs}PM_2)$ $d(\zeta(F_{A_1}), \zeta'(F_{A_2})) = d(\zeta'(F_{A_2}), \zeta(F_{A_1}))\ \forall\ \zeta(F_{A_1}), \zeta'(F_{A_2}) \widetilde{\in} (F_{\widetilde{R}})$.
$(\mathfrak{fpfs}PM_3)$ $d(\zeta(F_{A_1}), \zeta''(F_{A_3}))\ \widetilde{\leq}\ d(\zeta(F_{A_1}), \zeta'(F_{A_2})) + d(\zeta'(F_{A_2}), \zeta''(F_{A_3}))$
$\forall\ \zeta(F_{A_1}), \zeta'(F_{A_2}), \zeta''(F_{A_3}) \widetilde{\in} (F_{\widetilde{R}})$.

Definition 3.6. *A mapping $d : \mathfrak{fpfs}C(F_{\widetilde{R}}) \times \mathfrak{fpfs}C(F_{\widetilde{R}}) \to \mathbb{R}(A)^*$ is called a $\mathfrak{fpfs}$-quasi metric on $\mathfrak{fpfs}$-set $F_{\widetilde{R}}$ if d gratifies the following conditions:*

($\mathfrak{fpfs}QM_1$) $d(\zeta(F_{A_1}),\zeta'(F_{A_2}))=\overline{0}$ *if* $\zeta(F_{A_1})=\zeta'(F_{A_2})$ *or* $\zeta'(F_{A_2})\widetilde{\subset}\zeta(F_{A_1})$.
($\mathfrak{fpfs}QM_2$) $d(\zeta(F_{A_1}),\zeta''(F_{A_3}))\ \widetilde{\leq}\ d(\zeta(F_{A_1}),\zeta'(F_{A_2}))+d(\zeta'(F_{A_2}),\zeta''(F_{A_3}))$
$\forall\ \zeta(F_{A_1}),\zeta'(F_{A_2}),\zeta''(F_{A_3})\widetilde{\in}(F_{\widetilde{R}})$.

Definition 3.7. *Let* $(F_{\widetilde{R}},d)$ *be a* $\mathfrak{fpfs}$*-metric space and* F_A *be a non-empty* $\mathfrak{fpfs}$*-subset of* $F_{\widetilde{R}}$*. Then the mapping* $d_{F_A}:\mathfrak{fpfs}C(F_A)\times\mathfrak{fpfs}C(F_A)\to\mathbb{R}(A)^*$ *given by* $d_{F_A}(\zeta(F_{A_1}),\zeta'(F_{A_2}))=d(\zeta(F_{A_1}),\zeta'(F_{A_2}))\ \forall\ \zeta(F_{A_1}),\zeta'(F_{A_2})\widetilde{\in}F_A$ *is a* $\mathfrak{fpfs}$*-metric on* F_A*. This* $\mathfrak{fpfs}$*-metric* d_{F_A} *is known as the relative* $\mathfrak{fpfs}$*-metric induced on* F_A *by* d*. The* $\mathfrak{fpfs}$*-metric space* (F_A,d_{F_A}) *is called a* $\mathfrak{fpfs}$*-metric subspace or simply* $\mathfrak{fpfs}$*-subspace of the* $\mathfrak{fpfs}$*-metric space* $(F_{\widetilde{R}},d)$.

Definition 3.8. *Let* $(F_{\widetilde{R}},d)$ *be a* $\mathfrak{fpfs}$*-metric space . Let* $\zeta(F_A)$ *be a fixed* $\mathfrak{fpfs}$*-point of* $F_{\widetilde{R}}$ *and* F_B *be a non-null* $\mathfrak{fpfs}$*-subset of* $F_{\widetilde{R}}$*. The* $\mathfrak{fpfs}$*-distance of the* $\mathfrak{fpfs}$*-point* $\zeta(F_A)$ *from the* $\mathfrak{fpfs}$*-set* F_B *is denoted by* $d(\zeta(F_A),F_B)$ *and defined by*
$d(\zeta(F_A),F_B)=\inf\{d(\zeta(F_A),\zeta'(F_C))$: *for every* $\mathfrak{fpfs}$*-point* $\zeta'(F_C)$ *in* $F_B\}$. *In the case* $\zeta(F_A)$ *be a* $\mathfrak{fpfs}$*-point of* F_B*, we get*
$d(\zeta(F_A),F_B)=\inf\{d(\zeta(F_A),\zeta'(F_C))$: *for every* $\mathfrak{fpfs}$*-point* $\zeta'(F_C)$ *in* $F_B\}$
$=d(\zeta(F_A),\zeta(F_A))=\overline{0}\Rightarrow d(\zeta(F_A),F_B)=\overline{0}$.

Definition 3.9. *Let* $(F_{\widetilde{R}},d)$ *be a* $\mathfrak{fpfs}$*-metric space and* F_A *and* F_B *be two non-null* $\mathfrak{fpfs}$*-subsets of* $F_{\widetilde{R}}$*. The* $\mathfrak{fpfs}$*-distance between the* $\mathfrak{fpfs}$*-sets* F_A *and* F_B *is symbolized by* $d(F_A,F_B)$ *and is defined by*
$d(F_A,F_B)=\inf\{d(\zeta(F_{A_1}),\zeta'(F_{B_1}))$: *for every* $\zeta(F_{A_1})\widetilde{\in}F_A,\zeta'(F_{B_1})\widetilde{\in}F_B\}$. *Since* d *is symmetric so,* $d(F_A,F_B)=d(F_B,F_A)$.

Example 3.10. *Let* F_A *and* F_B *be two* $\mathfrak{fpfs}$*-sets of* $(F_{\widetilde{R}},d)$ *and* $F_A\widetilde{\cap}F_B\neq F_\phi$ *which implies that there exists* $\zeta(F_C)\widetilde{\in}F_A\widetilde{\cap}F_B$ *and*
$d(F_A,F_B)=\inf\{d(\zeta(F_{A_1}),\zeta'(F_{B_1}))$: *for every* $\zeta(F_{A_1})\widetilde{\in}F_A,\zeta'(F_{B_1})\widetilde{\in}F_B\}$
$d(F_A,F_B)=d(\zeta(F_C),\zeta(F_C))=\overline{0}$
$\therefore d(F_A,F_B)=\overline{0}$.
But converse is not necessarily true. It may so happen that $d(F_A,F_B)=\overline{0}$*, but* $F_A\widetilde{\cap}F_B=F_\phi$.

Definition 3.11. *A* $\mathfrak{fpfs}$*-metric space* $(F_{\widetilde{R}},d)$*is called* $\mathfrak{fpfs}$*-bounded if there exists a positive* $\mathfrak{fpfs}$*-real number* $\widetilde{k}$ *such that*
$d(\zeta(F_{A_1}),\zeta'(F_{A_2}))\widetilde{\leq}\widetilde{k}\ \forall\ \zeta(F_{A_1}),\zeta'(F_{A_2})\widetilde{\in}F_{\widetilde{R}}$.

Definition 3.12. *Let* $(F_{\widetilde{R}},d)$ *be a* $\mathfrak{fpfs}$*-metric space and* F_A *be a non-null* $\mathfrak{fpfs}$*-subset of* $F_{\widetilde{R}}$*. Then the* $\mathfrak{fpfs}$*-diameter of* F_A *is denoted as*
$\delta(F_A)=\sup\{d(\zeta(F_{A_1}),\zeta'(F_{A_2}))$: *for every* $\zeta(F_{A_1}),\zeta'(F_{A_2})\widetilde{\in}F_A\}$.

Remark. *In case, the supremum does not exists finitely for any* $\mathfrak{fpfs}$*-set* F_A *then we say that* $\mathfrak{fpfs}$*-set* F_A *is of infinite* $\mathfrak{fpfs}$*-diameter. It is obvious that for any non-null* $\mathfrak{fpfs}$*-set* F_A *of* $F_{\widetilde{R}}$ $\delta(F_A)\widetilde{\geq}\ \overline{0}$.

Theorem 3.13. *Let* $(F_{\widetilde{R}},d)$ *be a* $\mathfrak{fpfs}$*-metric space . Then*
(i) $\delta(F_A)=\overline{0}$ *if and only if* F_A *consists of a single* $\mathfrak{fpfs}$*-element.*
(ii) For every $\mathfrak{fpfs}$*-subsets* F_A,F_B *of* $F_{\widetilde{R}}$, $F_A\widetilde{\subset}F_B\Rightarrow\delta(F_A)\widetilde{\leq}\delta(F_B)$.
(iii) For every $\mathfrak{fpfs}$*-subsets* F_A,F_B *of* $F_{\widetilde{R}}$*, with* $F_A\widetilde{\cap}F_B\neq F_\phi$,

$$\delta(F_A\widetilde{\cap}F_B)\ \widetilde{\leq}\ \delta(F_A)+\delta(F_B)$$

Proof. (i) As given is that F_A consists of a single $\mathfrak{fpfs}$-element $\zeta(F_{A_1})$, then by definition $\delta(F_A) = \sup\{d(\zeta(F_{A_1}), \zeta(F_{A_1})) : \zeta(F_{A_1})\widetilde{\in}F_A\}$. Since $d(\zeta(F_{A_1}), \zeta(F_{A_1})) = \overline{0}$ if $\zeta(F_{A_1}) = \zeta(F_{A_1})$. This implies that $\delta(F_A) = \overline{0}$.
(ii) Let F_A and F_B be two $\mathfrak{fpfs}$-subsets of $F_{\widetilde{R}}$ and $F_A\widetilde{\subset}F_B$ then for every $\zeta(F_{A_1})\widetilde{\in}F_A$ implies that $\zeta(F_{A_1})\widetilde{\in}F_B$.
By definition of $\mathfrak{fpfs}$-diameters of a $\mathfrak{fpfs}$-set F_A, we can write that
$\delta(F_A) = \sup\{d(\zeta(F_{A_1}), \zeta'(F_{A_2})) :$ for every $\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_A\}$
$\widetilde{\leq}\sup\{d(\zeta(F_{B_1}), \zeta'(F_{B_2})) :$ for every $\zeta(F_{B_1}), \zeta'(F_{B_2})\widetilde{\in}F_B\} = \delta(F_B)$
$\Rightarrow \delta(F_A)\widetilde{\leq}\delta(F_B)$.
(iii) In case $F_A\widetilde{\subset}F_B$, then $F_A\widetilde{\cup}F_B = F_B$, then $\delta(F_A\widetilde{\cup}F_B) = \delta(F_B)$, so that $\delta(F_A\widetilde{\cup}F_B)\widetilde{\leq}\ \delta(F_A) + \delta(F_B)$, since $\delta(F_A)\widetilde{\geq}\overline{0}$. Similarly when $F_B\widetilde{\subset}F_A$, this proposition can be proved.
Next let us suppose that neither $F_A\widetilde{\subset}F_B$ nor $F_B\widetilde{\subset}F_A$. Let $\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_A\widetilde{\cup}F_B$, then $\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_A$ or $\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_B$.
We consider the following cases:
Case(I): Suppose $\zeta(F_{A_1}), \zeta'(F_{A_2})$ belongs to any one of the $\mathfrak{fpfs}$-sets F_A and F_B. If $\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_A$, then
$d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}\delta(F_A)$; if
$\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_B$, then $d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}\delta(F_B)$.
In either case
$d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}\ \delta(F_A) + \delta(F_B)$.
Case(II): Next we consider the case when any one of the $\mathfrak{fpfs}$-points belongs to F_A and another to the $\mathfrak{fpfs}$-set F_B. Without any lose of generality we can assume that $\zeta(F_{A_1})\widetilde{\in}F_A$, $\zeta'(F_{A_2})\widetilde{\in}F_B$.
Since $F_A\widetilde{\cap}F_B \neq F_\phi$, $\exists\zeta''(F_{A_3})\widetilde{\in}F_A\widetilde{\cap}F_B$.
Then by triangle inequality of d,
$d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}d(\zeta(F_{A_1}), \zeta''(F_{A_3})) + d(\zeta''(F_{A_3}), \zeta'(F_{A_2}))$.
So, $d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}\ \sup\{d(\zeta(F_{A_1}), \zeta''(F_{A_3})) : \zeta(F_{A_1}), \zeta''(F_{A_3})\widetilde{\in}F_A\}$
$+ \sup\{d(\zeta''(F_{A_3}), \zeta'(F_{A_2})) : \zeta''(F_{A_3}), \zeta'(F_{A_2})\widetilde{\in}F_B\} = \delta(F_A) + \delta(F_B)$.
Thus $d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}\ \delta(F_A) + \delta(F_B)$.
Thus we find for $\zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_A\widetilde{\cup}F_B$
$d(\zeta(F_{A_1}), \zeta'(F_{A_2}))\widetilde{\leq}\ \delta(F_A) + \delta(F_B)$.
So, $\sup\{d(\zeta(F_{A_1}), \zeta'(F_{A_2})) : \zeta(F_{A_1}), \zeta'(F_{A_2})\widetilde{\in}F_A\widetilde{\cup}F_B\}\widetilde{\leq}\ [\delta(F_A) + \delta(F_B)]$
i.e $\delta(F_A\widetilde{\cup}F_B)\widetilde{\leq}\ \delta(F_A) + \delta(F_B)$. □

Definition 3.14. *Let $(F_{\widetilde{R}}, d)$ be a $\mathfrak{fpfs}$-metric space and $\widetilde{r}$ be a non-negative $\mathfrak{fpfs}$-real number. For any $\mathfrak{fpfs}$-point $\zeta(F_A)\widetilde{\in}F_{\widetilde{R}}$, by a $\mathfrak{fpfs}$-open ball with center $\zeta(F_A)$ and radius $\widetilde{r}$, we mean the collection of $\mathfrak{fpfs}$-points of $F_{\widetilde{R}}$ satisfying*
$d(\zeta(F_B), \zeta(F_A))\widetilde{<}\widetilde{r}$; $\zeta(F_B)\widetilde{\in}F_{\widetilde{R}}$. *The $\mathfrak{fpfs}$-open ball with center $\zeta(F_A)$ and radius $\widetilde{r}$ is denoted by $\widetilde{B}(\zeta(F_A), \widetilde{r})$.*
Thus $\widetilde{B}(\zeta(F_A), \widetilde{r}) = \{\zeta(F_B)\widetilde{\in}F_{\widetilde{R}} :\ d(\zeta(F_B), \zeta(F_A))\widetilde{<}\ \widetilde{r}\ \widetilde{\subset}\mathfrak{fpfs}(F_{\widetilde{R}})\}$.
$\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ *will be called a $\mathfrak{fpfs}$-open ball with center $\zeta(F_A)$ and radius $\widetilde{r}$.*

Remark. *If $\mathbb{B}$ be a collection of $\mathfrak{fpfs}$-points then the $\mathfrak{fpfs}$-set generated by taking all the $\mathfrak{fpfs}$-points of $\mathbb{B}$ will be denoted by $\mathfrak{fpfs}(\mathbb{B})$; whereas the collection of all $\mathfrak{fpfs}$-points of a $\mathfrak{fpfs}$-set F_A will be expressed by $\mathfrak{fpfs}P(F_A)$.*

Definition 3.15. *Let $(F_{\widetilde{R}}, d)$ be a $\mathfrak{fpfs}$-metric space and $\widetilde{r}$ be a non-negative $\mathfrak{fpfs}$-real number. For any $\mathfrak{fpfs}$-point $\zeta(F_A)\widetilde{\in}F_{\widetilde{R}}$, by a $\mathfrak{fpfs}$-closed ball with center $\zeta(F_A)$*

and radius $\widetilde{r}$, we mean the collection of $\mathfrak{fpfs}$-points of $F_{\widetilde{R}}$ satisfying $d(\zeta(F_B),\zeta(F_A))\widetilde{\leq}\widetilde{r}$; $\zeta(F_B)\widetilde{\in}F_{\widetilde{R}}$. The $\mathfrak{fpfs}$-open ball with center $\zeta(F_A)$ and radius $\widetilde{r}$ is denoted by $\widetilde{B}[\zeta(F_A),\widetilde{r}]$.
Thus $\widetilde{B}[\zeta(F_A),\widetilde{r}]=\{\zeta(F_B)\widetilde{\in}F_{\widetilde{R}}:\ d(\zeta(F_B),\zeta(F_A))\widetilde{\leq}\ \widetilde{r}\ \widetilde{\subset}\mathfrak{fpfs}(F_{\widetilde{R}})\}$.
$\mathfrak{fpfs}(\widetilde{B}[\zeta(F_A),\widetilde{r}])$ will be called a $\mathfrak{fpfs}$-closed ball with center $\zeta(F_A)$ and radius $\widetilde{r}$.

Definition 3.16. *Let $(F_{\widetilde{R}},d)$ be a $\mathfrak{fpfs}$- metric space having at least two $\mathfrak{fpfs}$-points. Then $(F_{\widetilde{R}},d)$ is said to possess $\mathfrak{fpfs}$-Hausdorff property, if $\zeta(F_A)$ and $\zeta(F_B)$ are two $\mathfrak{fpfs}$-points in $F_{\widetilde{R}}$ in the way that $d(\zeta(F_A),\zeta(F_B))\widetilde{>}\overline{0}$, then there are two $\mathfrak{fpfs}$-open balls $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A),\widetilde{r_1}))$ and $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_B),\widetilde{r_2}))$ with centers $\zeta(F_A)$ and $\zeta(F_B)$ respectively and radius $\widetilde{r_1},\widetilde{r_2}\widetilde{>}\overline{0}$ such that*

$$\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A),\widetilde{r_1}))\widetilde{\cap}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_B),\widetilde{r_2}))=F_{\phi}.$$

Theorem 3.17. *Every $\mathfrak{fpfs}$-Metric space is $\mathfrak{fpfs}$-Hausdorff.*

Proof. Let $(F_{\widetilde{R}},d)$ be a $\mathfrak{fpfs}$-metric space having at least two $\mathfrak{fpfs}$-points. Let $\zeta(F_A)$ and $\zeta(F_B)$ be two $\mathfrak{fpfs}$-points in $F_{\widetilde{R}}$ in the manner that $d(\zeta(F_A),\zeta(F_B))\widetilde{>}\overline{0}$. Let us consider any $\mathfrak{fpfs}$-real number $\widetilde{r}$ satisfying $\overline{0}\ \widetilde{<}\ \widetilde{r}\ \widetilde{<}\ \frac{1}{2}d(\zeta(F_A),\zeta(F_B))$. Then $\widetilde{r}\widetilde{\in}\widetilde{\mathbb{R}}$ and the $\mathfrak{fpfs}$-open balls of radius $\widetilde{r}$ with centers $\zeta(F_A),\zeta(F_B)$ are $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A),\widetilde{r})=\mathfrak{fpfs}\{\zeta(F_C)\widetilde{\in}F_{\widetilde{R}}:d(\zeta(F_C),\zeta(F_A))\widetilde{<}\ \widetilde{r}\}$,
$\mathfrak{fpfs}(\widetilde{B}(\zeta(F_B),\widetilde{r})=\mathfrak{fpfs}\{\zeta(F_C)\widetilde{\in}F_{\widetilde{R}}:d(\zeta(F_C),\zeta(F_B))\widetilde{<}\ \widetilde{r}\}$.
We now prove that these two $\mathfrak{fpfs}$-open balls have void intersection.
If not, then there is some $\zeta(F_D)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A),\widetilde{r}))\widetilde{\cap}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_B),\widetilde{r}))$.
Now, $\zeta(F_D)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A),\widetilde{r}))$
$\Rightarrow d(\zeta(F_A),\zeta(F_D))\widetilde{<}\ \widetilde{r}$ and $\zeta(F_D)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_B),\widetilde{r}))$
$\Rightarrow d(\zeta(F_B),\zeta(F_D))\widetilde{<}\ \widetilde{r}$.
By $(\mathfrak{fpfs}M_4)$
$d(\zeta(F_A),\zeta(F_B))\ \widetilde{\leq}\ d(\zeta(F_A),\zeta(F_D))+d(\zeta(F_D),\zeta(F_B))\ \widetilde{<}\ \widetilde{r}+\widetilde{r}=2\widetilde{r}$
$\Rightarrow\widetilde{r}\ \widetilde{>}\ \frac{1}{2}d(\zeta(F_A),\zeta(F_B))$.
This is contradiction the hypothesis, so we must have
$\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A),\widetilde{r}))\widetilde{\cap}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_B),\widetilde{r}))=F_{\phi}$. Therefore $(F_{\widetilde{R}},d)$ is $\mathfrak{fpfs}$-Hausdorff space. □

Definition 3.18. *Let $(F_{\widetilde{R}},d)$ be a $\mathfrak{fpfs}$-metric space and $\zeta(F_A)\widetilde{\in}F_{\widetilde{R}}$. A collection $N(\zeta(F_A))$ of $\mathfrak{fpfs}$-points containing the $\mathfrak{fpfs}$-point $\zeta(F_A)$ is said to be $\mathfrak{fpfs}$-neighborhood of the $\mathfrak{fpfs}$-point $\zeta(F_A)$, if there subsists a positive $\mathfrak{fpfs}$-real number $\widetilde{r}$ such that $\zeta(F_A)\widetilde{\in}\widetilde{B}(\zeta(F_A),\widetilde{r}))\widetilde{\subset}N(\zeta(F_A))$.*
$\mathfrak{fpfs}(N(\zeta(F_A)))$ will be called a $\mathfrak{fpfs}$-neighborhood of the $\mathfrak{fpfs}$-point $\zeta(F_A)$.

Theorem 3.19. *Let $(F_{\widetilde{R}},d)$ be a $\mathfrak{fpfs}$-metric space and $\zeta(F_A)\widetilde{\in}F_{\widetilde{R}}$. Let N_1 and N_2 be $\mathfrak{fpfs}$-neighborhoods of $\zeta(F_A)$ in $(F_{\widetilde{R}},d)$. Then $\mathfrak{fpfs}(N_1)\widetilde{\cap}\mathfrak{fpfs}(N_2)$ is a $\mathfrak{fpfs}$-neighborhood of $\zeta(F_A)$ in $(F_{\widetilde{R}},d)$.*

Proof. Let $(F_{\widetilde{R}},d)$ be a $\mathfrak{fpfs}$-metric space and $\zeta(F_A)$ be any $\mathfrak{fpfs}$-point of $F_{\widetilde{R}}$. Let $N_1(\zeta(F_A))$ and $N_2(\zeta(F_A))$ be two $\mathfrak{fpfs}$-neighborhoods of $\zeta(F_A)$ in $(F_{\widetilde{R}},d)$. Then by definition for $N_1(\zeta(F_A))$ there exists $\widetilde{B}_1(\zeta(F_A),\widetilde{r_1})$ such that

$$\zeta(F_A)\widetilde{\in}\widetilde{B}_1(\zeta(F_A),\widetilde{r_1})\widetilde{\subset}N_1(\zeta(F_A))$$

and for $N_2(\zeta(F_A))$ there exists $\widetilde{B}_2(\zeta(F_A), \widetilde{r_2})$ such that

$$\zeta(F_A)\widetilde{\in}\widetilde{B}_2(\zeta(F_A), \widetilde{r_2})\widetilde{\subset}N_2(\zeta(F_A)).$$

If $\widetilde{B}_3(\zeta(F_A), \widetilde{r_3}) = \widetilde{B}_1(\zeta(F_A), \widetilde{r_1})\widetilde{\cap}\widetilde{B}_2(\zeta(F_A), \widetilde{r_2})$ and
$N_3(\zeta(F_A)) = N_1(\zeta(F_A))\widetilde{\cap}N_1(\zeta(F_A))$.
Then by using above three expressions we can write that

$$\zeta(F_A)\widetilde{\in}\widetilde{B}_3(\zeta(F_A), \widetilde{r_3})\widetilde{\subset}N_3(\zeta(F_A)).$$

Which implies that $\mathfrak{fpfs}(N_1)\widetilde{\cap}\mathfrak{fpfs}(N_2)$ is a $\mathfrak{fpfs}$-neighborhood of $\zeta(F_A)$ in $(F_{\widetilde{R}}, d)$. □

Theorem 3.20. *Every $\mathfrak{fpfs}$-open ball is a $\mathfrak{fpfs}$-neighborhood of each of its $\mathfrak{fpfs}$-points.*

Proof. Let $(F_{\widetilde{R}}, d)$ be a $\mathfrak{fpfs}$-metric space. Suppose that we have an $\mathfrak{fpfs}$-open ball $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ with center $\zeta(F_A)$ and radius $\widetilde{r}$. It is obvious that $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ is a $\mathfrak{fpfs}$-neighborhood of $\zeta(F_A)$ in $(F_{\widetilde{R}}, d)$.
Let an arbitrary $\mathfrak{fpfs}$-point $\zeta(F_B)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$, other than $\zeta(F_A)$. Thus we have,
$\overline{0} \neq d(\zeta(F_B), \zeta(F_A))\widetilde{<}\widetilde{r}$. Select any $\widetilde{r'}$ with $\overline{0} \widetilde{<} \widetilde{r'} \widetilde{<} \widetilde{r} - d(\zeta(F_B), \zeta(F_A))$. Then $\widetilde{r'}$ is a positive $\mathfrak{fpfs}$-number. Now for an arbitrary $\zeta(F_C)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B'}(\zeta(F_B), \widetilde{r'}))$, we have $d(\zeta(F_B), \zeta(F_C))\widetilde{<}\widetilde{r'}$. Now by ($\mathfrak{fpfs}M_4$), we get
$d(\zeta(F_A), \zeta(F_C))\widetilde{\leq} d(\zeta(F_A), \zeta(F_B)) + d(\zeta(F_B), \zeta(F_C))\widetilde{<} d(\zeta(F_A), \zeta(F_B)) + \widetilde{r'}\widetilde{<}\widetilde{r}$
$\therefore\ \zeta(F_C)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$
$\therefore\ \zeta(F_B)\widetilde{\in}\mathfrak{fpfs}(\widetilde{B'}(\zeta(F_B), \widetilde{r'}))\widetilde{\subset}\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$
$\Rightarrow \mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ is a $\mathfrak{fpfs}$-neighborhood of each of its $\mathfrak{fpfs}$-points. □

Theorem 3.21. *In a $\mathfrak{fpfs}$-metric space every $\mathfrak{fpfs}$-open ball is a $\mathfrak{fpfs}$-open set.*

Proof. Let $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ be $\mathfrak{fpfs}$-open ball with center $\zeta(F_A)$ and radius $\widetilde{r}$ in a $\mathfrak{fpfs}$-metric space $(F_{\widetilde{R}}, d)$. Clearly every $\mathfrak{fpfs}$-point of $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ is $\mathfrak{fpfs}$-interior points of $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$. Thus $\mathfrak{fpfs}(\widetilde{B}(\zeta(F_A), \widetilde{r}))$ is a $\mathfrak{fpfs}$-open set in the $\mathfrak{fpfs}$-metric space $(F_{\widetilde{R}}, d)$. □

4. Application of $\mathfrak{fpfs}$-metric space

Definition 4.1. *"Support of a $\mathfrak{fpfs}$-set F_A is a soft set over X delineated as*
$Supp(F_A) = \{(\zeta, F(\zeta)) : \mu_{F_A}(\zeta) \neq 0$ and $\gamma^{\zeta}_{F_A}(\vartheta) \neq 0\ \forall\ \zeta \in R,\ \vartheta \in X\}$

Example 4.2. *Let $X = \{\vartheta_1, \vartheta_2, \vartheta_3\}$ be the universal set and $R = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4\}$ be the set of parameters. If $A = \{\zeta_1, \zeta_2\} \subseteq R$ then $\mathfrak{fpfs}$-set F_A can be written as*

$$F_A = \{(0.3/\zeta_1, \{0.1/\vartheta_1, 0/\vartheta_2, 0.9/\vartheta_3\}), (0/\zeta_2, \{0.1/\vartheta_1, 0/\vartheta_2, 0.5/\vartheta_3\})\}$$

then by definition

$$Supp(F_A) = \{(\zeta_1, \{\vartheta_1, \vartheta_3\})\}$$

which is a soft set over X".

Example 4.3. *"Farming flowers is an appreciation for many people and for those who grow flower gardens for commercial intentions. Flowers are charming, resistless to birds, bees, wasps, other flower pollinators and even humans. There are several motives to spring up flowers. They appeal vultures that will help to wipe out*

the plagues in your garden. Flowers of different variety can make nitrogen and help inseminate your garden. They lead to seeds to replant your garden. They can help weed control".
Let a person Mr. R wants to grow some beautiful plants of flowers in his garden. The set of different flower plants is represented by $X = \{\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8\}$.
where
ψ_1=*Passion Flower,*
ψ_2=*Gazania,*
ψ_3=*Plumeria,*
ψ_4=*Chrysanthemum,*
ψ_5=*Rose,*
ψ_6=*Orchids,*
ψ_7=*Water lilies,*
ψ_8=*Tulip,*
ψ_9=*Dahlia.*
The set of attributes is given by $R = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6\}$, *where*
ζ_1 = *beautiful,*
ζ_2 = *vibrant,*
ζ_3 = *beautiful fragrance,*
ζ_4 = *low cost,*
ζ_5 = *long life,*
ζ_6 = *reproduction.*
***Algorithm**: We construct an algorithm for decision-making method by using support of a* $\mathfrak{fpfs}$*-set. The algorithm based on the following steps.*
Input:
***step 1:** Construct a* $\mathfrak{fpfs}$*-metric space* $(F_{\widetilde{R}}, d)$ *using universal set* X *and set of parameters* R.
***step 2:** Select some* $\mathfrak{fpfs}$*-open sets from the* $\mathfrak{fpfs}$*-metric space* $(F_{\widetilde{R}}, d)$.
Output:
***step 3:** Find the support of all selected* $\mathfrak{fpfs}$*-open sets.*
***step 4:** Find the choice values of all soft sets obtained from the support of* $\mathfrak{fpfs}$*-open sets by adding entries of each row separately.*
***step 5:** Find the final decision set by adding all choice values of obtained soft sets.*
***step 6:** Find the largest choice value from* $F_{decision}$.
By following the above algorithm we construct a $\mathfrak{fpfs}$*-metric space* $(F_{\widetilde{R}}, d)$ *on the given universal set* X *and the set of parameters* R. *Then we choose some* $\mathfrak{fpfs}$*-open balls from* $\mathfrak{fpfs}$*-metric space. It is clear from Theorem 3.21 that in* $\mathfrak{fpfs}$*-metric space every* $\mathfrak{fpfs}$*-open ball in a* $\mathfrak{fpfs}$*-open set, so to make our calculations easier we take the* $\mathfrak{fpfs}$*-open balls in the form of* $\mathfrak{fpfs}$*-open sets. The fuzzy subset* A *of* R *is given by* $A = \{0.9/\zeta_1, 0/\zeta_2, 0/\zeta_3, 0.7/\zeta_4, 0.5/\zeta_5, 0.8/\zeta_6\}$.
The $\mathfrak{fpfs}$*-set is given by*
$F_A = \{(0.95/\zeta_1, \{0.45/\psi_1, 0.64/\psi_2, 0.33/\psi_3, 0.22/\psi_4, 0/\psi_5, 0.71/\psi_6, 0/\psi_7, 0/\psi_8, 0.19/\psi_9\}),$
$(0.74/\zeta_4, \{0/\psi_1, 0/\psi_2, 0.35/\psi_3, 0/\psi_4, 0.64/\psi_5, 0/\psi_6, 0/\psi_7, 0.23/\psi_8, 0.12/\psi_9\}),$
$(0.53/\zeta_5, \{0.48/\psi_1, 0.67/\psi_2, 0/\psi_3, 0.26/\psi_4, 0.65/\psi_5, 0.74/\psi_6, 0/\psi_7, 0.23/\psi_8, 0/\psi_9\}),$
$(0.82/\zeta_6, \{0.48/\psi_1, 0/\psi_2, 0.37/\psi_3, 0.26/\psi_4, 0.65/\psi_5, 0/\psi_6, 0.84/\psi_7, 0.23/\psi_8, 0.12/\psi_9\})\}$.
In tabular form the $\mathfrak{fpfs}$*-set can be represented as,*

F_A	$0.95/\zeta_1$	$0/\zeta_2$	$0/\zeta_3$	$0.74/\zeta_4$	$0.53/\zeta_5$	$0.82/\zeta_6$
ψ_1	*0.45*	*0*	*0*	*0*	*0.48*	*0.48*
ψ_2	*0.64*	*0*	*0*	*0*	*0.67*	*0*
ψ_3	*0.33*	*0*	*0*	*0.35*	*0*	*0.37*
ψ_4	*0.22*	*0*	*0*	*0*	*0.26*	*0.26*
ψ_5	*0*	*0*	*0*	*0.64*	*0.65*	*0.65*
ψ_6	*0.71*	*0*	*0*	*0*	*0.74*	*0*
ψ_7	*0*	*0*	*0*	*0*	*0*	*0.84*
ψ_8	*0*	*0*	*0*	*0.23*	*0.23*	*0.23*
ψ_9	*0.19*	*0*	*0*	*0.12*	*0*	*0.12*

The tabular form of support of $\mathfrak{fpfs}$*-set* F_A *with choice values is given as*

SuppF_A	ζ_1	ζ_4	ζ_5	ζ_6	*choice value*
ψ_1	*1*	*0*	*1*	*1*	*3*
ψ_2	*1*	*0*	*1*	*0*	*2*
ψ_3	*1*	*1*	*0*	*1*	*3*
ψ_4	*1*	*0*	*1*	*1*	*3*
ψ_5	*0*	*1*	*1*	*1*	*3*
ψ_6	*1*	*0*	*1*	*0*	*2*
ψ_7	*0*	*0*	*0*	*1*	*1*
ψ_8	*0*	*1*	*1*	*1*	*3*
ψ_9	*1*	*1*	*0*	*1*	*3*

Similarly for the fuzzy sets $B = \{0/\zeta_1, 0.4/\zeta_2, 0.7/\zeta_3, 0/\zeta_4, 0.5/\zeta_5, 0/\zeta_6\}$ *and* $C = \{0.9/\zeta_1, 0/\zeta_2, 0.8/\zeta_3, 0/\zeta_4, 0/\zeta_5, 0.3/\zeta_6\}$ *of* R *the tabular forms of* $\mathfrak{fpfs}$*-sets* F_B *and* F_C *are*

F_B	$0/\zeta_1$	$0.45/\zeta_2$	$0.74/\zeta_3$	$0/\zeta_4$	$0.53/\zeta_5$	$0/\zeta_6$
ψ_1	*0*	*0*	*0.72*	*0*	*0.47*	*0*
ψ_2	*0*	*0.57*	*0*	*0*	*0.64*	*0*
ψ_3	*0*	*0*	*0.34*	*0*	*0*	*0*
ψ_4	*0*	*0.24*	*0*	*0*	*0.23*	*0*
ψ_5	*0*	*0.18*	*0.55*	*0*	*0.67*	*0*
ψ_6	*0*	*0*	*0.14*	*0*	*0*	*0*
ψ_7	*0*	*0.99*	*0*	*0*	*0.82*	*0*
ψ_8	*0*	*0*	*0.63*	*0*	*0.21*	*0*
ψ_9	*0*	*0.63*	*0*	*0*	*0*	*0*

F_C	$0.95/\zeta_1$	$0/\zeta_2$	$0.84/\zeta_3$	$0/\zeta_4$	$0/\zeta_5$	$0.33/\zeta_6$
ψ_1	*0*	*0*	*0*	*0*	*0*	*0.43*
ψ_2	*0.67*	*0*	*0*	*0*	*0*	*0.65*
ψ_3	*0.34*	*0*	*0*	*0*	*0*	*0.33*
ψ_4	*0.22*	*0*	*0*	*0*	*0*	*0*
ψ_5	*0.69*	*0*	*0.26*	*0*	*0*	*0*
ψ_6	*0.75*	*0*	*0.19*	*0*	*0*	*0*
ψ_7	*0.82*	*0*	*0*	*0*	*0*	*0.82*
ψ_8	*0*	*0*	*0*	*0*	*0*	*0.28*
ψ_9	*0.13*	*0*	*0.55*	*0*	*0*	*0.19*

The tabular form of support of $\mathfrak{fpfs}$*-set* F_B *with choice values is given as*

SuppF_B	ζ_2	ζ_3	ζ_5	*choice value*
ψ_1	*0*	*1*	*1*	*2*
ψ_2	*1*	*0*	*1*	*2*
ψ_3	*0*	*1*	*0*	*1*
ψ_4	*1*	*0*	*1*	*2*
ψ_5	*1*	*1*	*1*	*3*
ψ_6	*0*	*1*	*0*	*1*
ψ_7	*1*	*0*	*1*	*2*
ψ_8	*0*	*1*	*1*	*2*
ψ_9	*1*	*0*	*0*	*1*

The tabular form of support of $\mathfrak{fpfs}$*-set* F_B *with choice values is given as*

SuppF_C	ζ_1	ζ_3	ζ_6	*choice value*
ψ_1	*0*	*0*	*1*	*1*
ψ_2	*1*	*0*	*1*	*2*
ψ_3	*1*	*0*	*1*	*1*
ψ_4	*1*	*0*	*0*	*1*
ψ_5	*1*	*1*	*0*	*2*
ψ_6	*1*	*1*	*0*	*2*
ψ_7	*1*	*0*	*1*	*2*
ψ_8	*0*	*0*	*1*	*2*
ψ_9	*1*	*1*	*1*	*3*

The final decision table for all choice values of SuppF_A*,Supp*F_B *and Supp*F_C *is given as*

$F_{decision}$	CV_{SuppF_A}	CV_{SuppF_B}	CV_{SuppF_C}	*final choice value*
ψ_1	*3*	*2*	*1*	*6*
ψ_2	*2*	*2*	*2*	*6*
ψ_3	*3*	*1*	*1*	*5*
ψ_4	*3*	*2*	*1*	*6*
ψ_5	*3*	*3*	*2*	*8*
ψ_6	*2*	*1*	*2*	*5*
ψ_7	*1*	*2*	*2*	*5*
ψ_8	*3*	*2*	*2*	*7*
ψ_9	*3*	*1*	*3*	*7*

where CV stands for choice values.
"We can conclude easily from the above table that the highest score is 8*, scored by* ψ_5 *and the decision is in favor of selecting* ψ_5*= Rose. The second predilection goes to* ψ_8*= Tulip and* ψ_9*= Dahlia, similarly third predilection goes to* ψ_1*= Passion Flower,* ψ_2*= Gazania and* ψ_4*=Chrysanthemum and so on".*

5. Conclusion

We introduced the notion of $\mathfrak{fpfs}$-metric space by using $\mathfrak{fpfs}$-points. We presented some properties of $\mathfrak{fpfs}$-open ball, $\mathfrak{fpfs}$-closed ball, $\mathfrak{fpfs}$-diameter, $\mathfrak{fpfs}$-neighborhoods and $\mathfrak{fpfs}$-Hausdroff space. There is an ample range for advance exploration of $\mathfrak{fpfs}$-metric space and given application would be helpful for new researchers for their supplementary research work. The idea of $\mathfrak{fpfs}$-metric space can be broadened to $\mathfrak{fpfs}$-Banach spaces, $\mathfrak{fpfs}$-inner product spaces, $\mathfrak{fpfs}$-topological vector spaces.

Conflict of interest
The authors declare that they have no conflict of interest.
Competing interest
The authors declare that they have no competing interest.
Authors contributions
The authors contributed to each part of this paper equally. The authors read and approved the final manuscript.

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