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AN APPROACH TO SOFT FUNCTIONS

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Abstract. In this paper, using a more appreciate definition of a soft point, i.e. a soft point is a soft set (F, E) such that for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$, we present a new approach to soft functions in a interesting way, and introduce the concepts of soft continuous, soft open, soft closed, and soft homeomorfic functions in a very different way from the source existing in the literature. In the investigation we prove theorems related to these concepts and provide with examples, and counterexamples.

1. Introduction

In order to solve complicated problems in social sciences, economics, engineering and environment etc., we cannot use classical methods. The solutions of such problems involve the use of mathematical principles based on imprecision. Thus classical set theory, which is based on the crisp and exact case may not be fully suitable for handling such problems of uncertainty. A number of theories have been proposed for dealing with uncertainties in an efficient way. Some of these are theories of fuzzy sets [23], (see also [16], [9], [7]), intuitionistic fuzzy sets [3], interval mathematics [11] and rough sets [21]. However, these theories have their own difficulties. Molodtsov [20] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties which is free from the above limitations (see [15, 17, 8] for a comprehensive review of the theory of soft sets). Later on, people have observed that soft sets have many applications in both pure and applied sciences. Topological studies on soft sets were started by M.Shabir and M.Naz [22] in which they introduced soft topological spaces, and some concepts of soft sets on soft topological spaces. Later, researches about soft topological spaces were studied in [4, 6, 10, 12, 13, 24] where the concept of a soft point is expressed by different approaches.

The purpose of this paper is to introduce, and investigate the concepts of soft continuous, soft open, soft closed, and soft homeomorfic functions, based on the soft point definition in [4], and present characterization theorems.

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2. Preliminaries

In this section, we recall some necessary definitions, and the results which will be needed in the paper. Throughout the paper X and E will denote an initial universe set and a set of parameters, respectively. By A we will always denote a subset of E, i.e. $A \subset E$.

Definition 1. *([20]) A pair (F, E) is called a soft set over X if only if F is a mapping from E into the set of all subsets of the set X, i.e., $F : E \to P(X)$, where $P(X)$ is the power set of X.*

Definition 2. *([15]) A soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $e \in A$, $F(e) = \emptyset$ (null set).*

Definition 3. *([15]) A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A$, $F(e) = X$.*

Definition 4. [15] *Let (F, A) and (G, B) be two soft sets over X . Then (F, A) is called a soft subset of (G, B), denoted by $(F, A) \subset (G, B)$, if (i) $A \subset B$, and (ii) $F(e) \subset G(e)$ for each $e \in A$.*

We note that the definition given by Maji et al. [15] seems not to be applied. In fact Irfan et al. [1] also gave the definition of a restricted intersection.

Definition 5. *([1]) The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C)*

$$H(e) = \begin{cases} F(e), \text{ if } e \in A - B \\ G(e), \text{ if } e \in B - A \\ F(e) \cap G(e), \text{ if } e \in A \cap B \end{cases}$$

, where $C = A \cap B$ and $\forall e \in C$.

Definition 6. *([15]) The union of two soft sets (F, A) and (G, B) over X is the soft set,*

$$H(e) = \begin{cases} F(e), \text{ if } e \in A - B \\ G(e), \text{ if } e \in B - A \\ F(e) \cup G(e), \text{ if } e \in A \cap B \end{cases}.$$

where $C = A \cup B$ and $\forall e \in C$, and is denoted by $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 7. *([22]) The complement of a soft set (F, E) is denoted by $(F, E)^c$ and is defined by $(F, E)^c = (F^c, E)$ where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in E$.*

Definition 8. *([22]) Let τ be a collection of soft sets over X. Then τ is said to be a soft topology on X if*
(1) Φ, $\tilde{X}$ belong to τ,
(2) the union of any number of soft sets in τ belongs to τ,
(3) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 9. *([22]) Let X be an initial universe set, E be the set of parameters and $\tau = \{\Phi, \tilde{X}\}$. The τ is called soft indiscrete topology on X and (X, τ, E) is said to be a soft indiscrete topological space over X.*

Definition 10. *([22]) Let X be an initial universe set, E be the set of parameters and let τ be the collection of all soft sets which can be defined over X. Then τ is*

called the soft discrete topology on X and (X, τ, E) is said to be an soft discrete topological space over X.

Definition 11. *([22]) Let (X, τ, E) be a soft topological space over X. A soft set (F, E) over X is said to be soft closed in X, if its complement $(F, E)^c$ belongs to τ.*

Proposition 2.1. *([22]) Let (X, τ, E) be a soft topological space over X. Then the collection $\tau_e = \{F(e) | (F, E) \in \tau\}$ defines a topology on X for each $e \in E$.*

Definition 12. *([22]) Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $\overline{(F, E)}$, is the intersection of all soft closed super sets of (F, E).*

Definition 13. *([22]) Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then we associate with a soft set (F, E) over X, denoted by $(\overline{F}, E)$ and defined as $\overline{F}(e) = \overline{F(e)}$, where $\overline{F(e)}$ is the closure of $F(e)$ in τ_e for each $e \in E$.*

Proposition 2.2. *([22]) Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then $(\overline{F}, E) \subset \overline{(F, E)}$.*

Corollary 2.3. *([22]) Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then $(\overline{F}, E) = \overline{(F, E)}$ if and only if $(\overline{F}, E)^c \in \tau$.*

Definition 14. *([13]) Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and $x \in X$. Then (G, E) is said to be a soft neighbourhood of x, if there exists a soft open set (F, E) such that $x \in (F, E) \subset (G, E)$.*

Definition 15. *([13]) Let (X, τ, E) be a soft topological space over X. Then soft interior of a soft set (F, E) over X is denoted by $(F, E)^\circ$ and is defined as the union of all soft open sets contained in (F, E). Thus $(F, E)^\circ$ is the largest soft open set contained in (F, E).*

3. Soft Continuity

Based on the the definition of a soft point given in [4] we introduce a different kind of a definition of soft continuity, and give a comprehensive investigation of the soft continuous mappings.

Definition 16. *([4]) Let (F, E) be a soft set over X, and $x \in X$. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$.*

Definition 17. *Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : (X, \tau, E) \to (Y, \tau', E)$ be a mapping. For each soft neighbourhood (H, E) of $(f(x)_e, E)$, if there exists a soft neighbourhood (F, E) of (x_e, E) such that $f((F, E)) \subset (H, E)$, then f is said to be a soft continuous mapping at (x_e, E).*
If f is soft continuous for all (x_e, E), then f is called soft continuous on X.

In the following theorem we give characterizations of soft continuity.

Theorem 3.1. *Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : (X, \tau, E) \to (Y, \tau', E)$ be a mapping. Then the following conditions are equivalent:*
(1) $f : (X, \tau, E) \to (Y, \tau', E)$ is a soft continuous mapping,

(2) For each soft open set (G,E) over Y, $f^{-1}((G,E))$ is a soft open set over X,
(3) For each soft closed set (H,E) over Y, $f^{-1}((H,E))$ is a soft closed set over X,
(4) For each soft set (F,E) over X, $f(\overline{(F,E)}) \subset \overline{(f(F,E))}$,
(5) For each soft set (G,E) over Y, $\overline{(f^{-1}(G,E))} \subset f^{-1}(\overline{(G,E)})$,
(6) For each soft set (G,E) over Y, $f^{-1}((G,E)^\circ) \subset (f^{-1}(G,E))^\circ$.

Proof. (1) $\Rightarrow$ (2) Let (G,E) be a soft open set over Y and $(x_e,E) \in f^{-1}(G,E)$ be an arbitrary soft point. Then $f(x_e,E) = (f(x)_e,E) \in (G,E)$. Since f is soft continuous mapping, there exists $(x_e,E) \in (F,E) \in \tau$ such that $f(F,E) \subset (G,E)$. This implies that $(x_e,E) \in (F,E) \subset f^{-1}(G,E)$, $f^{-1}((G,E))$ is a soft open set over X.
(2) $\Rightarrow$ (1) Let (x_e,E) be a soft point and $(f(x)_e,E) \in (G,E)$ be an arbitrary soft neighbourhood. Then $(x_e,E) \in f^{-1}(G,E)$ is a soft neighborhood and $f(f^{-1}(G,E)) \subset (G,E)$.
(3) $\Rightarrow$ (4) Let (F,E) be a soft set over X. Since $(F,E) \subset f^{-1}(f(F,E))$ and $f(F,E) \subset \overline{(f(F,E))}$, we have $(F,E) \subset f^{-1}(f(F,E)) \subset f^{-1}\overline{(f(F,E))}$. By part (3), since $f^{-1}\overline{(f(F,E))}$ is a soft closed set over X, $\overline{(F,E)} \subset f^{-1}\overline{(f(F,E))}$. Thus $f(\overline{(F,E)}) \subset f(f^{-1}\overline{(f(F,E))}) \subset \overline{f(F,E)}$ is obtained.
(4) $\Rightarrow$ (5) Let (G,E) be a soft set over Y and $f^{-1}(G,E) = (F,E)$. By part (4), we have $f(\overline{(F,E)}) = f(\overline{f^{-1}(G,E)}) \subset \overline{f(f^{-1}(G,E))} \subset \overline{(G,E)}$. Then $\overline{f^{-1}(G,E)} = \overline{(F,E)} \subset f^{-1}(f\overline{(F,E)}) \subset f^{-1}(\overline{(G,E)})$.
(5) $\Rightarrow$ (6) Let (G,E) be a soft set over Y. Substituting $(G,E)^c$ for condition in (5). Then $\overline{f^{-1}((G,E)^c)} \subset f^{-1}(\overline{(G,E)^c})$. Since $(G,E)^\circ = \overline{((G,E)^c)}^c$, then we have $f^{-1}((G,E)^\circ) = f^{-1}(\overline{((G,E)^c)}^c) = (f^{-1}(\overline{(G,E)^c}))^c \subset (\overline{f^{-1}((G,E)^c)})^c = (\overline{(f^{-1}(G,E))^c})^c = (f^{-1}(G,E))^\circ$.
(6) $\Rightarrow$ (2) Let (G,E) be a soft open set over Y. Since $(f^{-1}(G,E))^\circ \subset f^{-1}(G,E) = f^{-1}((G,E)^\circ) \subset (f^{-1}(G,E))^\circ$, then $(f^{-1}(G,E))^\circ = f^{-1}(G,E)$ is obtained. This implies that $f^{-1}(G,E)$ is a soft open set over X. □

Example 1. *Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E), (F_2,E)\}$, $\tau' = \{\Phi, \tilde{X}, (G_1,E), (G_2,E)\}$ be two soft topologies defined on X, where (F_1,E), (F_2,E), (G_1,E) and (G_2,E) are soft sets over X, defined as follows:*

$$F_1(e_1) = \{h_1,h_2\}, \quad F_1(e_2) = \{h_3\}, \quad F_2(e_1) = X, \quad F_2(e_2) = \{h_3\},$$

and

$$G_1(e_1) = \{h_1\}, \quad G_1(e_2) = \{h_3\}, \quad G_2(e_1) = \{h_1,h_3\}, \quad G_2(e_2) = \{h_2,h_3\},$$

If we get the mapping $f : X \to X$ defined as

$$f(h_1) = f(h_2) = h_1, \ f(h_3) = h_3$$

then since $f^{-1}(G_1,E) = (F_1,E)$ and $f^{-1}(G_2,E) = (F_2,E)$, f is a soft continuous mapping.

Example 2. *Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1,E), (F_2,E), (F_3,E), (F_4,E)\}$, $\tau' = \{\Phi, \tilde{X}, (G_1,E), (G_2,E), (G_3,E), (G_4,E)\}$ be two soft topologies defined on X where $(F_1,E), (F_2,E), (F_3,E), (F_4,E), (G_1,E), (G_2,E), (G_3,E)$ and (G_4,E) are soft sets over X, defined as follows:*

$$F_1(e_1) = \{h_2\}, \quad F_1(e_2) = \{h_1\}, \quad F_2(e_1) = \{h_2,h_3\}, \quad F_2(e_2) = \{h_1,h_2\},$$

$$F_3(e_1) = \{h_3\}, \quad F_3(e_2) = \{h_1, h_2\}, \quad F_4(e_1) = \emptyset, \quad F_4(e_2) = \{h_1\},$$
$$F_5(e_1) = X, \quad F_2(e_2) = \{h_1, h_2\}$$

and

$$G_1(e_1) = \{h_2\}, \quad G_1(e_2) = \{h_1\}, \quad G_2(e_1) = \{h_2, h_3\}, \quad G_2(e_2) = \{h_1, h_2\},$$
$$G_3(e_1) = \{h_1, h_2\}, \quad G_3(e_2) = X, \quad G_4(e_1) = \{h_2\}, \quad G_4(e_2) = \{h_1, h_2\},$$

Then (X, τ, E), (X, τ', E) are two soft topological spaces and $f = 1_X : X \to X$ is not a soft continuous mapping, where 1_X is identical function.

Example 3. *Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : (X, \tau, E) \to (Y, \tau', E)$ be a mapping. If τ' is the soft indiscrete topology on Y, then f is a soft continuous mapping.*

Example 4. *Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : (X, \tau, E) \to (Y, \tau', E)$ be a mapping. If τ is a soft discrete topology on X, then f is a soft continuous mapping.*

Theorem 3.2. *If $f : (X, \tau, E) \to (Y, \tau', E)$ is a soft continuous mapping, then for each $e \in E$, $f_e : (X, \tau_e) \to (Y, \tau'_e)$ is a continuous mapping.*

Proof. Let $U \in \tau'_e$. Then there exists a soft open set (G, E) over Y such that $U = G(e)$. Since $f : (X, \tau, E) \to (Y, \tau', E)$ is a soft continuous mapping, $f^{-1}(G, E)$ is a soft open set over X and $f^{-1}(G, E)(e) = f^{-1}(G(e)) = f^{-1}(U)$ is an open set. This implies that f_e is a continuous mapping. □

Now we give an example to show that the converse of the above theorem does not hold.

Example 5. *Let $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$ and $E = \{e_1, e_2\}$. Then $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ is a soft topological space over X and $\tau' = \{\Phi, \tilde{Y}, (G_1, E), (G_2, E), (G_3, E)\}$ is a soft topological space over Y. Here $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ are soft sets over X and $(G_1, E), (G_2, E), (G_3, E)$ are soft sets over Y, defined as follows:*

$$F_1(e_1) = \{x_1\}, \quad F_1(e_2) = \{x_1, x_3\}, \quad F_2(e_1) = \{x_2\}, \quad F_2(e_2) = \{x_1\},$$
$$F_3(e_1) = \{x_1, x_2\}, \quad F_3(e_2) = \{x_1, x_3\}, \quad F_4(e_1) = \emptyset, \quad F_4(e_2) = \{x_1\}, \quad F_5(e_1) = \{x_1, x_2\}, \quad F_5(e_2) = X.$$

and

$$G_1(e_1) = Y, \quad G_1(e_2) = \{y_2\}, \quad G_2(e_1) = \{y_1\}, \quad G_2(e_2) = \{y_2\}, \quad G_3(e_1) = \{y_1, y_2\}, \quad G_3(e_2) = \{y_2\}.$$

If we consider the mapping $f : X \to Y$ defined as

$$f(x_1) = y_2, \ f(x_2) = y_1, \ f(x_3) = y_3$$

, then f is not a soft continuous mapping, since $f^{-1}(G_1, E) \notin \tau$, where $f^{-1}(G_1)(e_1) = X$, $f^{-1}(G_1)(e_2) = \{x_1\}$. Also, $f_{e_1} : (X, \tau_{e_1}) \to (Y, \tau_{e_1})$ and $f_{e_2} : (X, \tau_{e_2}) \to (Y, \tau_{e_2})$ are continuous mappings. Here

$$\tau_{e_1} = \{\emptyset, X, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}, \quad \tau_{e_2} = \{\emptyset, X, \{x_1\}, \{x_1, x_3\}\}$$

and

$$\tau'_{e_1} = \{\emptyset, Y, \{y_1\}, \{y_1, y_2\}\}, \quad \tau'_{e_2} = \{\emptyset, Y, \{y_2\}\}.$$

The converse of the above theorem is also true when the complement of the soft set $(\overline{F}, E)$ is soft open over X, for each soft set (F, E).

Theorem 3.3. *If $(\overline{F}, E)^c$ is a soft open set over X, for each soft set (F, E), then $f : (X, \tau, E) \to (Y, \tau', E)$ is a soft continuous mapping if and only if $f_e : (X, \tau_e) \to (Y, \tau'_e)$ is a continuous mapping, for each $e \in E$.*

Proof. Let $f_e : (X, \tau_e) \to (Y, \tau'_e)$ be a continuous mapping, for each $e \in E$, and let (F, E) be an arbitrary soft set over X. Then $f_e(\overline{F}(e)) \subset \overline{f(F)}(e)$ is satisfied, for each $e \in E$. Since $(\overline{F}, E)^c \in \tau$, $(\overline{F}, E) = \overline{(F, E)}$ from Corollary 1. Thus $f(\overline{(F, E)}) \subset \overline{f((F, E))}$ is obtained. This implies that $f : (X, \tau, E) \to (Y, \tau', E)$ is a soft continuous mapping. □

Theorem 3.4. *Let (X, τ, E) and $(Y, \tau', E)$and(Z, τ^*, E) be soft topological spaces. If $f : (X, \tau, E) \to (Y, \tau', E)$ and $g : (Y, \tau', E) \to (Z, \tau^*, E)$ are soft continuous mappings, then $gof : (X, \tau, E) \to (Z, \tau^*, E)$ is a soft continuous mapping.*

Proof. Let $(W, E) \in \tau^*$ be a soft open set and let us show that $(gof)^{-1}(W, E)$ is a soft open set in X. Since $(gof)^{-1}(W, E) = f^{-1}(g^{-1}(W, E))$ and the mapping g is a soft continuous mapping, then $g^{-1}(W, E)$ is a soft open set in Y. On the other hand, since f is a soft continuous mapping, then $f^{-1}(g^{-1}(W, E))$ is a soft open set in X. That is, $(gof)^{-1}(W, E)$ is a soft open in X and gof is a soft continuous mapping. □

4. Soft Homeomorphism

Now we first give the following definitions.

Definition 18. *Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : X \to Y$ be a mapping.*
a) If the image $f((F, E))$ of each soft open set (F, E) over X is a soft open set in Y, then f is said to be a soft open mapping.
b) If the image $f((H, E))$ of each soft closed set (H, E) over X is a soft closed set in Y, then f is said to be a soft closed mapping.

Proposition 4.1. *If $f : (X, \tau, E) \to (Y, \tau', E)$ is soft open (closed), then for each $e \in E$, $f_e : (X, \tau_e) \to (Y, \tau'_e)$ is an open (closed) mapping.*

Proof. The proof of the proposition is straightforward and it is left to the reader. □

Note that the concepts of soft continuous, soft open and soft closed mappings are all independent of each other.

Example 6. *Let (X, τ, E) be soft discrete topological space and (X, τ', E) be soft indiscrete topological space. Then $1_X : (X, \tau, E) \to (X, \tau', E)$ is a soft open and soft closed mapping. But it is not a soft continuous mapping.*

Example 7. *Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), ..., (F_7, E)\}$, $\tau' = \{\Phi, \tilde{X}, (G_1, E), (G_2, E), (G_3, E), (G_4, E)\}$ be two soft topologies defined on X where $(F_1, E), (F_2, E), (F_3, E), ..., (F_7, E), (G_1, E), (G_2, E), (G_3, E)$ and (G_4, E) are soft sets over X, defined as follows:*

$$F_1(e_1) = \{h_2\}, \quad F_1(e_2) = \{h_1\}, \quad F_2(e_1) = \{h_1, h_3\}, \quad F_2(e_2) = \{h_2, h_3\},$$
$$F_3(e_1) = \{h_2\}, \quad F_3(e_2) = X, \quad F_4(e_1) = \emptyset, \quad F_4(e_2) = \{h_1\},$$
$$F_5(e_1) = \{h_1, h_3\}, \quad F_5(e_2) = X, \quad F_6(e_1) = \emptyset, \quad F_6(e_2) = \{h_2, h_3\},$$
$$F_7(e_1) = \emptyset, \quad F_7(e_2) = X$$

and

$$G_1(e_1) = \{h_2\}, \quad G_1(e_2) = \{h_1\}, \quad G_2(e_1) = \{h_2, h_3\}, \quad G_2(e_2) = \{h_1, h_2\},$$
$$G_3(e_1) = \{h_1, h_2\}, \quad G_3(e_2) = X, \quad G_4(e_1) = \{h_2\}, \quad G_4(e_2) = \{h_1, h_2\}.$$

If we consider the mapping $f : X \to X$ defined as $f(h_i) = h_1$, for $1 \leq i \leq 3$. It is clear that
$f^{-1}(G_1)(e_1) = f^{-1}(G_2)(e_1) = f^{-1}(G_4)(e_1) = \emptyset$,
$f^{-1}(G_1)(e_2) = f^{-1}(G_2)(e_2) = f^{-1}(G_4)(e_2) = X$,
$f^{-1}(G_3)(e_1) = X$, $f^{-1}(G_3)(e_2) = X$. *Then f is a soft continuous mapping, but*

$$f(F_1)(e_1) = \{h_1\}, \ \ f(F_1)(e_2) = \{h_1\}, \ \ f(F_1^c)(e_1) = \{h_1\}, \ \ f(F_1^c)(e_2) = \{h_1\}.$$

Hence it is neither soft open nor soft closed.

Example 8. *Let $X = \{h_1, h_2, h_3\}$, $Y = \{a, b\}$ and $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E)\}$, $\tau' = \{\Phi, \tilde{Y}, (G_1, E), (G_2, E)\}$ be two soft topologies defined on X and Y, respectively. Here $(F_1, E), (F_2, E), (G_1, E), (G_2, E)$ are soft sets over X and Y, respectively. The soft sets are defined as follows:*

$$F_1(e_1) = \{h_1, h_2\}, \quad F_1(e_2) = \{h_3\}, \quad F_2(e_1) = X, \quad F_2(e_2) = \{h_3\},$$

and

$$G_1(e_1) = Y, \quad G_1(e_2) = \{b\}, \quad G_2(e_1) = \{a\}, \quad G_2(e_2) = \{b\},$$

If we get the mapping $f : X \to Y$ defined as

$$f(h_1) = \{a\}, \quad f(h_2) = f(h_3) = \{b\}.$$

It is clear that

$$f(F_1)(e_1) = Y, \ \ f(F_1)(e_2) = \{b\}, \ \ f(F_2)(e_1) = Y, \ \ f(F_2)(e_2) = \{b\}.$$

Then the mapping $f : X \to Y$ is a soft open mapping. Also since $f(F_1^c)(e_1) = \{b\}$, $f(F_1^c)(e_2) = Y$, it is not soft closed mapping and $f^{-1}(G_1)(e_1) = X$, $f^{-1}(G_1)(e_2) = \{h_2, h_3\}$. Hence it is not a soft continuous mapping.

Example 9. *Let $X = \{h_1, h_2, h_3\}$, $Y = \{a, b\}$, $E = \{e_1, e_2\}$ and $\tau = \{\Phi, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$, $\tau' = \{\Phi, \tilde{Y}, (G_1, E), (G_2, E)\}$ be two soft topologies defined on X and Y, respectively. Here $(F_1, E), (F_2, E), (F_3, E), (G_1, E), (G_2, E)$ are soft sets over X and Y, respectively. The soft sets are defined as follows:*

$$F_1(e_1) = \{h_1, h_3\}, \ F_1(e_2) = \{h_2\}, \ F_2(e_1) = X, \ F_2(e_2) = \{h_2, h_3\}, \ F_3(e_1) = \{h_3\}, \ F_3(e_2) = \{h_2\}$$

and

$$G_1(e_1) = \{b\}, \quad G_1(e_2) = \emptyset, \quad G_2(e_1) = Y, \quad G_2(e_2) = \{b\}.$$

Now we define the mapping $f : X \to Y$ as $f(h_1) = f(h_2) = \{a\}$, $f(h_3) = \{b\}$. It is clear that $f(F_1^c(e_1)) = f(h_2) = \{a\}, f(F_1^c(e_2)) = f(\{h_1, h_3\}) = Y$, $f(F_2^c(e_1)) = \emptyset, f(F_2^c(e_2)) = f(\{h_1\}) = \{a\}$, $f(F_3^c(e_1)) = \{a\}, f(F_3^c(e_2)) = Y$. This implies that f is a soft closed mapping. Also $f(F_1(e_1)) = Y, f(F_1(e_2)) = \{a\}$, $f^{-1}(G_1(e_1)) = \emptyset, f^{-1}(G_1(e_2)) = \{h_1, h_2\}$. Then it is neither soft open, nor soft continuous.

Theorem 4.2. *Let (X, τ, E) and (Y, τ', E) be two soft topological spaces, $f : X \to Y$ be a mapping.*
a) f is a soft open mapping if and only if for each soft set (F, E) over X, $f((F, E)^\circ) \subset (f(F, E))^\circ$ is satisfied.
b) f is a soft closed mapping if and only if for each soft set (F, E) over X, $\overline{(f(F, E))} \subset f(\overline{(F, E)})$ is satisfied.

Proof. a) Let f be a soft open mapping and (F,E) be a soft set over X. $(F,E)^\circ$ is a soft open set and $(F,E)^\circ \subset (F,E)$. Since f is a soft open mapping, $f((F,E)^\circ)$ is a soft open set in Y and $f((F,E)^\circ) \subset f((F,E))$. Thus $f((F,E)^\circ) \subset f((F,E))^\circ$ is obtained. Conversely, let (F,E) be any soft open set over X. Then $(F,E) = (F,E)^\circ$. From the condition of theorem, we have $f((F,E)^\circ) \subset (f(F,E))^\circ$. Then $f((F,E)) = f((F,E)^\circ) \subset (f(F,E))^\circ \subset f((F,E))$. This implies that $f((F,E)) = (f(F,E))^\circ$. This completes the proof.
b) Let f be a soft closed mapping and (F,E) be any soft set over X. Since f is a soft closed mapping, $f(\overline{(F,E)})$ is a soft closed set over Y and $f((F,E)) \subset f(\overline{(F,E)})$. Thus $\overline{f(F,E)} \subset f(\overline{(F,E)})$ is obtained. Conversely, let (F,E) be any soft closed set over X. From the condition of theorem, $\overline{(f(F,E))} \subset f(\overline{(F,E)}) = f((F,E)) \subset \overline{(f(F,E))}$. This means that $\overline{(f(F,E))} = f((F,E))$. This completes the proof. □

Definition 19. *Let (X,τ,E) and (Y,τ',E) be two soft topological spaces, $f : X \to Y$ be a mapping. If f is a bijection, soft continuous and f^{-1} is a soft continuous mapping, then f is said to be a soft homeomorphism from X to Y. When a homeomorphism f exists between X and Y, we say that X is soft homeomorphic to Y.*

Theorem 4.3. *Let (X,τ,E) and (Y,τ',E) be two soft topological spaces, $f : X \to Y$ be a bijective mapping. Then the following conditions are equivalent:*
(1) f is a soft homeomorphism,
(2) f is a soft continuous and soft closed mapping,
(3) f is a soft continuous and soft open mapping.

Proof. The proofs can be easily obtained by using the theorems on continuity, openness, and closedness, so are omitted. □

5. Conclusion

In this paper, introducing the concepts of soft continuity, soft opennes, soft closedness, and soft homeomorphism based on the notions given in [4] which are different from those of [6, 10, 12, 13, 14, 18, 19, 24] , further results are given some characterization theorems are obtained, and supported with examples and counterexamples. We expect that our introduced notion and investigation might be a reference for further studies, so that one may expect it to be a more useful tool in the field of soft set theory in modeling various problems occurring in many areas of science, computer science, smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, economics, game theory, operations research, and in many kinds of real life problems.

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