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SOME NEW IDENTITIES OF SYMMETRY FOR CARLITZ'S-TYPE TWISTED q-TANGENT POLYNOMIALS UNDER S_4

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Abstract. The primary purpose of this paper is to investigate some new and interesting symmetric properties for the Carlitz's-type twisted q-Tangent polynomials arising from the fermionic p-adic integral on $\mathbb{Z}_p$ under S_4.

1. Introduction

In the complex plane, the Euler polynomials are defined by the generating function below:

$$\frac{2}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!},\ (|t|<\pi).$$

When $x=0$, we get $E_n(0):=E_n$ which is called the n-th Euler number, see [4], [6], [8], [9], [10].

Numerous properties of Bernoulli, Euler, Genocchi and Tangent numbers and polynomials possess many applications in analytic number theory, special functions, physics, combinatorics and the other interested fields. In spite of their being already century old, aforementioned numbers and polynomials are still today enveloped in an aura of enigma within scientific community. These numbers and polynomials, and many kinds of their generalizations have been extensively studied and investigated by many mathematicians introducing new techniques to get not only new but also interesting identities for above polynomials and numbers and their various generalizations (see [1-10]). The quantum calculus, denoted by q-calculus, in the area of special functions have been developed in the last two decades (see [1-5,7,8,10]). Various generalizations of special functions based on q-numbers were given by means of p-adic q-integral on $\mathbb{Z}_p$ (or q-Volkenborn integral) firstly introduced by Kim [5]. q-Volkenborn integral over the p-adic numbers field is a strong tool in order to establish many generating functions of special polynomials, and so as to derive old and new properties and formulae. In this paper, in particular, we are interested in the generalizations of Tangent polynomials and numbers arising from fermionic p-adic integral on $\mathbb{Z}_p$, see, e.g., [6,8,9].

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As well-known that the Tangent numbers T_{2n-1} $(n \geq 1)$ are defined as the coefficients of the Taylor expansion of $\tan x$:

$$\tan x = \sum_{n=1}^{\infty} \frac{T_{2n-1}}{(2n-1)!} x^{2n-1}, \text{ see [6,9]}.$$

Originally, the following relation holds true for familiar Euler numbers and usual Tangent numbers:

$$E_{2n-1} = (-1)^n \frac{T_{2n-1}}{2^{2n-1}}. \tag{1.1}$$

Ryoo [9] introduced Tangent-type polynomial $\widetilde{T}_n(x)$ which is different from original definition, as follows:

$$\sum_{n=0}^{\infty} \widetilde{T}_n(x) \frac{t^n}{n!} = \frac{2}{e^{2t}+1} e^{xt}, \qquad (|t| < \frac{\pi}{2}). \tag{1.2}$$

Letting $x = 0$ in the Eq. (1.2) reduces to $\widetilde{T}_n(0) := \widetilde{T}_n$ that is called n-th Tangent-type number (see, e.g., [8], [9]).

Ryoo's Tangent polynomials satisfy the following equality (see [9])

$$E_{2n-1} = \frac{\widetilde{T}_{2n-1}}{2^{2n-1}}. \tag{1.3}$$

Notice that the Eqs. (1.1) and (1.3) are different. Further we obtain

$$\widetilde{T}_{2n-1} = (-1)^n T_{2n-1}. \tag{1.4}$$

Because of (1.4), we call $\widetilde{T}_n(x)$ and $\widetilde{T}_n$ as Tangent-type polynomials and Tangent-type numbers rather than Tangent polynomials and Tangent numbers.

Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of p-adic rational integers, the field of rational numbers, the field of p-adic rational numbers and the completion of algebraic closure of $\mathbb{Q}_p$, respectively, where p be a fixed odd prime number. Let $\mathbb{N} = \{1, 2, 3, \cdots\}$ and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$.

Supposing d is an odd positive number with $(d, p) = 1$. Let

$$X := X_d = \lim_{\overleftarrow{n}} \mathbb{Z}/dp^N\mathbb{Z} \text{ and } X_1 = \mathbb{Z}_p$$

and

$$a + dp^N\mathbb{Z}_p = \left\{ x \in X / x \equiv a \left(moddp^N \right) \right\}$$

where $a \in \mathbb{Z}$ lies in $0 \leq a < dp^N$ (see [1-5,7-9]).

The normalized absolute value according to the theory of p-adic analysis is given by $|p|_p = p^{-1}$. The notation "q" can be considered as an indeterminate, a complex number $q \in \mathbb{C}$ with $|q| < 1$, or a p-adic number $q \in \mathbb{C}_p$ with $|q-1|_p < p^{-\frac{1}{p-1}}$ and $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. It is always clear in the content of the paper.

The q-numbers are defined as

$$[x]_q = \frac{1-q^x}{1-q} \text{ and } [x]_{-q} = \frac{1-(-q)^x}{1+q}.$$

It is clear that $\lim_{q \to 1} [x]_q = x$ (for details, see [1-5,7-10]).

For

$$f \in UD(\mathbb{Z}_p) = \{ f \,|\, f : \mathbb{Z}_p \to \mathbb{C}_p \text{ is uniformly differentiable function} \},$$

the fermionic p-adic integral on $\mathbb{Z}_p$ of a function $f \in UD(\mathbb{Z}_p)$ is defined by the following equality:

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_{-1}(x) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x, \tag{1.5}$$

which satisfy the following property:

$$I_{-1}(f_n) = (-1)^n I_{-1}(f) + 2\sum_{r=0}^{n-1} (-1)^{n-l-1} f(r),$$

where $f_n(x)$ means $f(x+n)$. For elaborated information, see, e.g., [3], [4], [8], [9].

Let $T_p = \bigcup_{N\geq 1} C_{p^N} = \lim_{N\to\infty} C_{p^N}$, where $C_{p^N} = \left\{w : w^{p^N} = 1\right\}$ is the cyclic group of order p^N. For $w \in T_p$, we indicate by $\phi_w : \mathbb{Z}_p \to C_p$ the locally constant function $x \to w^x$. For $q \in C_p$ with $|1-q|_p < 1$ and $w \in T_p$, the Carlitz's-type twisted q-Tangent polynomials are defined by the following p-adic fermionic integral on $\mathbb{Z}_p$, with respect to μ_{-1}, in [8]:

$$\int_{\mathbb{Z}_p} w^y q^y [2y+x]_q^n\, d\mu_{-1}(y) = \widetilde{T}_{n,q,w}(x) \quad (n \geq 0). \tag{1.6}$$

Substituting $x = 0$ in the Eq. (1.6) gives $\widetilde{T}_{n,q,w}(0) := \widetilde{T}_{n,q,w}$ that are called Carlitz's-type twisted q-Tangent numbers. These numbers can be generated by the following recurrence relation:

$$qw(q^2\widetilde{T}_{q,w} + [2]_q)^n + \widetilde{T}_{n,q,w} = \begin{cases} 2, & \text{if } n = 0 \\ 0, & \text{if } n \neq 0 \end{cases}$$

with technque method notation by replacing $\left(\widetilde{T}_{q,w}\right)^n$ by $\widetilde{T}_{n,q,w}$, symbolically.

Letting $q \to 1^-$ and $w = 1$ in the Eq. (1.6) gives

$$\widetilde{T}_n(x) := \int_{\mathbb{Z}_p} (2y+x)^n d\mu_{-1}(y).$$

Symmetric identities of some special polynomials associated with p-adic q-integrals on $\mathbb{Z}_p$ have been studied and investigated by many authors (see [1-5,7-9]). Duran *et al.* [1] derived some new symmetric identities of q-Genocchi polynomials derived from the q-Volkenborn integral on $\mathbb{Z}_p$. Duran *et al.* [2] obtained some new symmetric identities of weighted q-Genocchi polynomials using q-Volkenborn integral on $\mathbb{Z}_p$. Araci *et al.* [4] derived some new symmetric identities for q-Frobenius Euler polynomials under S_5 which are derived from the fermionic p-adic integral over the p-adic numbers field. Kim *et al.* [7] obtained some new identities of symmetry for Carlitz's q-Bernoulli polynomials under symmetric group of degree five. Kim *et al.* [6] presented new identities of symmetry for Carlitz's-type q-Bernoulli polynomials by using the q-Volkenborn integrals on $\mathbb{Z}_p$ under S_5.

2. Some new identities of symmetry for $\widetilde{T}_{n,q,w}(x)$ under S_4

In this section, we derive some new and interesting symmetric identities for Carlitz's-type twisted q-Tangent polynomials associated with the fermionic p-adic integral on $\mathbb{Z}_p$ under the symmetric group of degree four denoted by S_4.

Let $w_i \in \mathbb{N}$ with $w_i \equiv 1(mod2)$ with $i \in \{1,2,3,4\}$. By the Eqs. (1.5) and (1.6), we attain

$$\int_{\mathbb{Z}_p} w^{w_1w_2w_3y} q^{w_1w_2w_3y} e^{[w_1w_2w_3 2y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t} d\mu_{-1}(y)$$

$$= \lim_{N\to\infty} \sum_{y=0}^{p^N-1} (-1)^y w^{w_1w_2w_3y} q^{w_1w_2w_3y} e^{[w_1w_2w_3 2y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t}$$

$$= \lim_{N\to\infty} \sum_{l=0}^{w_4-1} \sum_{y=0}^{p^N-1} (-1)^{l+y} w^{w_1w_2w_3(l+w_4y)} q^{w_1w_2w_3(l+w_4y)}$$

$$\times e^{[w_1w_2w_3 2(l+w_4y)+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t}.$$

Taking

$$\sum_{i=0}^{w_1-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_3-1} (-1)^{i+j+k} w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} q^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k}$$

on the both sides of the above yields to

$$I = \sum_{i=0}^{w_1-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_3-1} (-1)^{i+j+k} w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} q^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k}$$

$$\times \int_{\mathbb{Z}_p} w^{w_1w_2w_3y} q^{w_1w_2w_3y} e^{[w_1w_2w_3 2y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t} d\mu_{-1}(y)$$

$$= \lim_{N\to\infty} \sum_{i=0}^{w_1-1} \sum_{j=0}^{w_2-1} \sum_{k=0}^{w_3-1} \sum_{l=0}^{w_4-1} \sum_{y=0}^{p^N-1} (-1)^{i+j+k+y+l}$$

$$\times w^{w_1w_2w_3(l+w_4y)+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} q^{w_1w_2w_3(l+w_4y)+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k}$$

$$\times e^{[w_1w_2w_3 2(l+w_4y)+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t}.$$

Note that Eq. (2) is invariant for any permutation $\sigma \in S_4$. Therefore, we acquire the following theorem.

Theorem 2.1. *Let $w_i \in \mathbb{N}$ with $w_i \equiv 1(mod2)$ with $i \in \{1,2,3,4\}$. Then the following equality*

$$I = \sum_{i=0}^{w_{\sigma(1)}-1} \sum_{j=0}^{w_{\sigma(2)}-1} \sum_{k=0}^{w_{\sigma(3)}-1} (-1)^{i+j+k} w^{w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k}$$

$$\times q^{w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k}$$

$$\times \int_{\mathbb{Z}_p} w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}(l+w_{\sigma(4)}y)} q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}(l+w_{\sigma(4)}y)}$$

$$\times e^{\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}2y+w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}x+w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k\right]_q t} d\mu_{-1}(y)$$

holds true for any $\sigma \in S_4$.

Using the definition of q-number, $[x]_q$, we easily see that

$$\begin{aligned} & [w_1w_2w_32y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q \quad (2.1) \\ = & [w_1w_2w_3]_q \left[2y+w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right]_{q^{w_1w_2w_3}}. \end{aligned}$$

By utilizing Eq. (2.1), we calculate

$$\begin{aligned} & \int_{\mathbb{Z}_p} w^{w_1w_2w_3y} q^{w_1w_2w_3y} e^{[w_1w_2w_32y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q t} d\mu_{-1}(y) \quad (2.2) \\ & = \sum_{n=0}^{\infty} [w_1w_2w_3]_q^n \left(\int_{\mathbb{Z}_p} w^{w_1w_2w_3y} q^{w_1w_2w_3y} \left[2y+w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right]^n_{q^{w_1w_2w_3}} d\mu_{-1}(y)\right)\frac{t^n}{n!} \\ & = \sum_{n=0}^{\infty} [w_1w_2w_3]_q^n \widetilde{T}_{n,q^{w_1w_2w_3},w^{w_1w_2w_3}} \left(w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right)\frac{t^n}{n!}. \end{aligned}$$

From Eq. (2.2), we obtain

$$\begin{aligned} & \int_{\mathbb{Z}_p} w^{w_1w_2w_3y} q^{w_1w_2w_3y} [w_1w_2w_32y+w_1w_2w_3w_4x+w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k]_q \, d\mu_{-1}(y) \quad (2.3) \\ & = [w_1w_2w_3]_q^n \widetilde{T}_{n,q^{w_1w_2w_3},w^{w_1w_2w_3}} \left(w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right), \text{ for } n \geq 0. \end{aligned}$$

Thus, from Theorem 2.1 and Eq. (2.3), we get the following theorem.

Theorem 2.2. *For $n \geq 0$, $w_i \in \mathbb{N}$ with $w_i \equiv 1(mod2)$ in conjunction with $i \in \{1,2,3,4\}$, the following equation*

$$\begin{aligned} I = & \left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}\right]_q^n \sum_{i=0}^{w_{\sigma(1)}-1}\sum_{j=0}^{w_{\sigma(2)}-1}\sum_{k=0}^{w_{\sigma(3)}-1} (-1)^{i+j+k} \\ & \times w^{w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k} q^{w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j+w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k} \\ & \times \widetilde{T}_{n,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}},w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}}} \left(w_{\sigma(4)}x+\frac{w_{\sigma(4)}}{w_{\sigma(1)}}i+\frac{w_{\sigma(4)}}{w_{\sigma(2)}}j+\frac{w_{\sigma(4)}}{w_{\sigma(3)}}k\right) \end{aligned}$$

holds true for any $\sigma \in S_4$.

Using binomial theorem and the definitions of q-number of x, the following equality can be derived with the simple calculations:

$$\begin{aligned} & \left[2y+w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right]^n_{q^{w_1w_2w_3}} \quad (2.4) \\ = & \sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_4]_q}{[w_1w_2w_3]_q}\right)^{n-m} [w_2w_3i+w_1w_3j+w_1w_2k]^{n-m}_{q^{w_4}} \\ & \times q^{m(w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k)} [2y+w_4x]^m_{q^{w_1w_2w_3}}. \end{aligned}$$

Applying $\int_{\mathbb{Z}_p} w^{w_1w_2w_3y}q^{w_1w_2w_3y}d\mu_{-1}(y)$ on the both sides of the above gives

$$\begin{aligned}
&\int_{\mathbb{Z}_p} w^{w_1w_2w_3y}q^{w_1w_2w_3y}\left[2y+w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right]^n_{q^{w_1w_2w_3}} d\mu_{-1}(y) \quad (2.5)\\
=&\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_4]_q}{[w_1w_2w_3]_q}\right)^{n-m}[w_2w_3i+w_1w_3j+w_1w_2k]^{n-m}_{q^{w_4}}\\
&\times q^{m(w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k)}\int_{\mathbb{Z}_p} w^{w_1w_2w_3y}q^{w_1w_2w_3y}\left[2y+w_4x\right]^m_{q^{w_1w_2w_3}}d\mu_{-1}(y)\\
=&\sum_{m=0}^{n}\binom{n}{m}\left(\frac{[w_4]_q}{[w_1w_2w_3]_q}\right)^{n-m}[w_2w_3i+w_1w_3j+w_1w_2k]^{n-m}_{q^{w_4}}\\
&\times q^{m(w_2w_3w_4i+w_1w_3w_4j+w_1w_2w_4k)}\widetilde{T}_{m,q^{w_1w_2w_3},w^{w_1w_2w_3}}(w_4x).
\end{aligned}$$

By the Eq. (2.5), we get

$$\begin{aligned}
&[w_1w_2w_3]^n_q\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}(-1)^{i+j+k}w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k}q^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k} \quad (2.6)\\
&\times\int_{\mathbb{Z}_p} w^{w_1w_2w_3y}q^{w_1w_2w_3y}\left[2y+w_4x+\frac{w_4}{w_1}i+\frac{w_4}{w_2}j+\frac{w_4}{w_3}k\right]^n_{q^{w_1w_2w_3}} d\mu_{-1}(y)\\
&=\sum_{m=0}^{n}\binom{n}{m}[w_1w_2w_3]^m_q[w_4]^{n-m}_q\widetilde{T}_{m,q^{w_1w_2w_3},w^{w_1w_2w_3}}(w_4x)\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\\
&\sum_{k=0}^{w_3-1}(-1)^{i+j+k}w^{w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k}q^{(m+1)(w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k)}\\
&\qquad\times[w_2w_3i+w_1w_3j+w_1w_2k]^{n-m}_{q^{w_4}}\\
&=\sum_{m=0}^{n}\binom{n}{m}[w_1w_2w_3]^m_q[w_4]^{n-m}_q\widetilde{T}_{m,q^{w_1w_2w_3},w^{w_1w_2w_3}}(w_4x)\,C_{n,q^{w_4},w^{w_4}}(w_1,w_2,w_3\mid m),
\end{aligned}$$

where

$$\begin{aligned}
&C_{n,q,w}(w_1,w_2,w_3\mid m) \quad (2.7)\\
=&\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}(-1)^{i+j+k}w^{w_2w_3i+w_1w_3j+w_1w_2k}\\
&\times q^{(m+1)(w_2w_3i+w_1w_3j+w_1w_2k)}[w_2w_3i+w_1w_3j+w_1w_2k]^{n-m}_q.
\end{aligned}$$

Therefore, by (2.7), we arrive at the following theorem.

Theorem 2.3. *Let $w_i\in\mathbb{N}$ with $w_i\equiv 1(mod 2)$ in conjunction with $i\in\{1,2,3,4\}$, and let $n\geq 0$. Then the following expression*

$$\begin{aligned}
&\sum_{m=0}^{n}\binom{n}{m}\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}\right]^m_q\left[w_{\sigma(4)}\right]^{n-m}_q\\
&\times\widetilde{T}_{m,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}},w^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}}}\left(w_{\sigma(4)}x\right)C_{n,q^{w_{\sigma(4)}},w^{w_{\sigma(4)}}}(w_{\sigma(1)},w_{\sigma(2)},w_{\sigma(3)}\mid m)
\end{aligned}$$

holds true for some $\sigma\in S_4$.

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