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I- LACUNARY GENERALIZED DIFFERENCE CONVERGENT SEQUENCES IN n-NORMED SPACES

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Abstract. In this paper we introduce I-lacunary convergence of generalized difference sequences by using a sequence of moduli in n-normed space.

1. Introduction

The concept of linear 2-normed spaces was initially developed by Gahler in [3, 4] in the 1960's, while that of n-normed spaces can be found in [3]. Since then, many authors including Gunawan [5, 6], Gunawan and Mashadi [7], Sahiner [13] have studied these concepts and obtained various results.

Let $n \in N$ and X be a real vector space of dimension d, where $n \leq d$. A real valued function $\|.,.,...,.\|$ on X^n satisfying the following four conditions:
(1) $\|x_1, x_2, ..., x_n\| = 0$ if and only if $x_1, x_2, ..., x_n$ are linearly dependent,
(2) $\|x_1, x_2, ..., x_n\|$ is invariant under permutation,
(3) $\|\alpha x_1, x_2, ..., x_n\| = |\alpha| \|x_1, x_2, ..., x_n\|$ for any $\alpha \in R$,
(4) $\|x + x', x_2, ..., x_n\| \leq \|x, x_2, ..., x_n\| + \|x', x_2, ..., x_n\|$
is called an n-norm on X and the pair $(X, \|.,.,...,.\|)$ is called an n-normed space.

A trivial example of an n-normed space is $X = R^n$ equipped with the following Euclidean n-norm:

$$\|x_1, x_2, ..., x_n\|_E = abs \left(\begin{vmatrix} x_{11} & . & . & . & x_{1n} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ x_{n1} & . & . & . & x_{nn} \end{vmatrix} \right),$$

where $x_i = (x_{i1}, \cdots, x_{in}) \in R^n$ for each $i = 1, 2, \cdots, n$. If $(X, \|.,.,...,.\|)$ be an n-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, \cdots, a_n\}$ be a linearly independent set in X. Then the following function $\|.,.,...,.\|_\infty$ on X^{n-1} defined by

$$\|x_1, x_2, ..., x_{n-1}\|_\infty = \max \{\|x_1, x_2, ..., x_{n-1}, a_i\| : i = 1, 2, \cdots, n\}$$

defines an $(n-1)$ norm on X with respect to $\{a_1, a_2, \cdots, a_n\}$ (see for details [7]).

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The standard n-norm on X, a real inner product space of dimension $d \geq n$ is as follows:

$$\left\| x_1, x_2, ... x_n \right\|_S = \begin{vmatrix} \langle x_1, x_1 \rangle & . & . & . & \langle x_1, x_n \rangle \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ \langle x_n, x_1 \rangle & . & . & . & \langle x_n, x_n \rangle \end{vmatrix}^{\frac{1}{2}},$$

where $\langle ., . \rangle$ denotes the inner product on X. If $X = R^n$, then this n-norm is exactly the same as the Euclidean n-norm $\left\| x_1, x_2, ..., x_n \right\|_E$ mentioned earlier. For $n = 1$, this n-norm is the usual norm $\left\| x \right\| = \langle x_1, x_1 \rangle^{\frac{1}{2}}$. Now we state the following important results as Lemmas which can be found in [7].

Lemma 1.1. *Every n-normed space is an $(n - r)$ normed space for all $r = 1, 2, \cdots, n - 1$. In particular, every n-normed space is a normed space.*

Lemma 1.2. *A standard n-normed space is complete if and only if it is complete with respect to usual norm $\left\| x \right\| = \langle x_1, x_1 \rangle^{\frac{1}{2}}$.*

Lemma 1.3. *On a standard n-normed space X, the derived $(n-1)$-norm $\left\| ., ..., . \right\|_\infty$ defined with respect to orthonormal set $\{e_1, e_2, \cdots, e_n\}$, is equivalent to the standard $(n - 1)$ norm $\left\| ., ..., . \right\|_S$. Precisely, we have*

$$\left\| x_1, ..., x_{n-1} \right\|_\infty \leq \left\| x_1, ..., x_{n-1} \right\|_S \leq \sqrt{n} \left\| x_1, ..., x_{n-1} \right\|_\infty$$

for all $x_1, x_2, \cdots, x_{n-1} \in X$, where

$$\left\| x_1, ..., x_{n-1} \right\|_\infty = \max \left\{ \left\| x_1, ..., x_{n-1}, e_i \right\|_S : i = 1, 2, \cdots, n \right\}.$$

By an ideal we mean a family $I \subset 2^X$ of subsets a non-empty set X satisfying: (i) $\phi \in I$, (ii) $A, B \in I$ imply $A \cup B \in I$, (iii) $A \in I$, $B \subset A$ imply $B \in I$, while an admissible ideal I of X further satisfies $\{y\} \in I$ for each $y \in X$ [9]. By lacunary sequence we mean an increasing sequence $\theta = (k_r)$ of positive integers satisfying; $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. We denote the intervals, which θ determines, by $I_r = (k_{r-1}, k_r]$.

We recall that modulus function is a function $f : [0, \infty) \to [0, \infty)$ such that
(i) $f(x) = 0$ if and only if $x = 0$,
(ii) $f(x + y) \leq f(x) + f(y)$ for all $x, y \geq 0$,
(iii) f is increasing,
(iv) f is continuous from the right at zero.
It follows from (i) and (iv) that f must be continuous everywhere on $[0,\infty)$.
For a sequence of moduli $\Omega = (f_k)$, we give the following conditions.
(v) $\sup_k f_k(x) < \infty$ for all $x > 0$,
(vi) $\lim_{x \to 0} f_k(x) = 0$ uniformly in $k \geq 1$.

We remark that in case $f_k = f$ for all k, where f is a modulus, the conditions (v) and (vi) are automatically fullfilled.

In [8], Kizmaz defined the sequence spaces $Z(\Delta) = \{x = (x_k) : (\Delta x_k) \in Z\}$ for $Z = l_\infty, c$ and c_0, where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$. After, Colak and Et [1] defined generalized the difference sequence spaces as follows: $Z(\Delta^m) = \{x = (x_k) : (\Delta^m x_k) \in Z\}$ for $Z = l_\infty$, c and c_0, where $m \in \mathbb{N}$, $\Delta^0 x = x_k$, $\Delta x = (x_k - x_{k+1})$, $\Delta^m x_k = (\Delta^m x_k) = \left(\Delta^{m-1} x_k - \Delta^{m-1} x_{k+1}\right)$ and so that

$$\Delta^m x_k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} x_{k+v}.$$

In this article, using lacunary sequences and the notion of ideal, we aimed to introduce some new generalized difference sequence spaces with respect to a sequence of moduli in n-normed linear spaces.

2. Definitions and Inclusions Theorems

Let I be an admissible ideal, $\Omega = (f_k)$ be a sequence of moduli, $(X, \|.,...,.\|)$ be an n-normed space, and $p = (p_k)$ be a sequence of positive real numbers. By $w(n-X)$ we denote the space of all sequences defined over n-normed space $(X, \|.,...,.\|)$. We define

$$[N_\theta, \Omega, \Delta^m, p, \|.,...,.\|, X_s]^I = \left\{ \begin{array}{l} (x_k) \in w(n-X): \\ \left[r \in \mathbb{N}: \ h_r^{-1} \sum_{k \in I_r} [f_k(\|\Delta^m x_k - L, z_1, z_2, ..., z_{n-1}\|)]^{p_k} \geq \varepsilon, \right. \\ \text{for some } L \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \ ] \in I \end{array} \right\}$$

$$[N_\theta, \Omega, \Delta^m, p, \|.,...,.\|, X_s]_0^I = \left\{ \begin{array}{c} (x_k) \in w(n-X): \\ \left[r \in \mathbb{N}: \ h_r^{-1} \sum_{k \in I_r} [f_k(\|\Delta^m x_k, z_1, z_2, ..., z_{n-1}\|)]^{p_k} \geq \varepsilon, \right. \\ \text{for every } z_1, z_2, ..., z_{n-1} \in X \ ] \in I \end{array} \right\}.$$

When $m = 0$ we obtain the following sequence spaces which were defined by Esi and Acikgoz [2].

$$[N_\theta, \Omega, p, \|.,...,.\|, X_s]^I = \left\{ \begin{array}{l} (x_k) \in w(n-X): \\ \left[r \in \mathbb{N}: \ h_r^{-1} \sum_{k \in I_r} [f_k(\|x_k - L, z_1, z_2, ..., z_{n-1}\|)]^{p_k} \geq \varepsilon, \right. \\ \text{for some } L \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \ ] \in I \end{array} \right\}$$

$$[N_\theta, \Omega, p, \|.,...,.\|, X_s]_0^I = \left\{ \begin{array}{c} (x_k) \in w(n-X): \\ \left[r \in \mathbb{N}: \ h_r^{-1} \sum_{k \in I_r} [f_k(\|x_k, z_1, z_2, ..., z_{n-1}\|)]^{p_k} \geq \varepsilon, \right. \\ \text{for every } z_1, z_2, ..., z_{n-1} \in X \ ] \in I \end{array} \right\}.$$

When $m = 1$, we obtain the following difference sequence spaces:

$$[N_\theta, \Omega, \Delta, p, \|.,...,.\|, X_s]^I = \left\{ \begin{array}{l} (x_k) \in w(n-X): \\ \left[r \in \mathbb{N}: \ h_r^{-1} \sum_{k \in I_r} [f_k(\|\Delta x_k - L, z_1, z_2, ..., z_{n-1}\|)]^{p_k} \geq \varepsilon, \right. \\ \text{for some } L \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \ ] \in I \end{array} \right\}$$

$$[N_\theta, \Omega, \Delta, p, \|.,...,.\|, X_s]_0^I = \left\{ \begin{array}{c} (x_k) \in w(n-X): \\ \left[r \in \mathbb{N}: \ h_r^{-1} \sum_{k \in I_r} [f_k(\|\Delta x_k, z_1, z_2, ..., z_{n-1}\|)]^{p_k} \geq \varepsilon, \right. \\ \text{for every } z_1, z_2, ..., z_{n-1} \in X \ ] \in I \end{array} \right\}.$$

Now we begin.

Theorem 2.1. *If $p = (p_k)$ is bounded, then*

$$[N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I$$

and

$$[N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]_0^I$$

are linear spaces over the complex field $\mathbb{C}$.

Proof. We consider only $[N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]_0^I$, the other can be treated similarly. If $H = \sup_k p_k$ and $K = \max\left(1, 2^{H-1}\right)$, we have Maddox([10], page 346)

$$|a_k + b_k|^{p_k} \leq K.\left(|a_k|^{p_k} + |b_k|^{p_k}\right). \tag{2.1}$$

Suppose that $x, y \in [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]_0^I$. For $\alpha, \beta \in \mathbb{C}$, there exists M_α and N_β integers such that $|\alpha| \leq M_\alpha$ and $|\beta| \leq N_\beta$. Since $\|., ..., .\|$ is a n-norm and f_k is an modulus function for all k and from (2.1), the following inequality holds:

$$\begin{aligned} & h_r^{-1} \sum_{k \in I_r} \left[f_k\left(\left\|\Delta^m\left(\alpha x_k + \beta y_k\right), z_1, z_2, ..., z_{n-1}\right\|\right)\right]^{p_k} \\ & \leq K.\left(M_\alpha\right)^H.h_r^{-1} \sum_{k \in I_r} \left[f_k\left(\left\|\Delta^m x_k, z_1, z_2, ..., z_{n-1}\right\|\right)\right]^{p_k} \\ & + K.\left(N_\beta\right)^H.h_r^{-1} \sum_{k \in I_r} \left[f_k\left(\left\|\Delta^m y_k, z_1, z_2, ..., z_{n-1}\right\|\right)\right]^{p_k}. \end{aligned}$$

On the other hand from the above inequality, we get

$$\begin{aligned} & \left\{r \in \mathbb{N} : h_r^{-1} \sum_{k \in I_r} \left[f_k\left(\left\|\Delta^m\left(\alpha x_k + \beta y_k\right), z_1, z_2, ..., z_{n-1}\right\|\right)\right]^{p_k} \geq \varepsilon\right\} \\ & \subseteq \left\{r \in \mathbb{N} : K.\left(M_\alpha\right)^H.h_r^{-1} \sum_{k \in I_r} \left[f_k\left(\left\|\Delta^m x_k, z_1, z_2, ..., z_{n-1}\right\|\right)\right]^{p_k} \geq \varepsilon\right\} \\ & \cup \left\{r \in \mathbb{N} : K.\left(N_\beta\right)^H.h_r^{-1} \sum_{k \in I_r} \left[f_k\left(\left\|\Delta^m y_k, z_1, z_2, ..., z_{n-1}\right\|\right)\right]^{p_k} \geq \varepsilon\right\}. \end{aligned}$$

Two sets on the right side belongs to I, so this completes the proof. □

Lemma 2.2. *Let f be a modulus function and let $0 < \delta < 1$. Then for each $x > \delta$ we have $f(x) \leq 2f(1)\delta^{-1}x$ [12].*

Theorem 2.3. *Let $\Omega = (f_k)$ be a sequence of moduli and $0 < \inf_k p_k = h \leq p_k \leq \sup_k p_k = H < \infty$. Then*

$$[N_\theta, \Delta^m, p, \|., ..., .\|, X_s]^I \subset [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I$$

and

$$[N_\theta, \Delta^m, p, \|., ..., .\|, X_s]_0^I \subset [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]_0^I.$$

Proof. If $x \in [N_\theta, \Delta^m, p, \|., ..., .\|, X_s]^I$, then for some $L > 0$ and for every $z_1, z_2, ..., z_{n-1} \in X$

$$\left\{r \in \mathbb{N} : h_r^{-1} \sum_{k \in I_r} \left(\left\|\Delta^m x_k - L, z_1, z_2, ..., z_{n-1}\right\|\right)^{p_k} \geq \varepsilon\right\}.$$

Now let $\varepsilon > 0$ be given. We can choose $0 < \delta < 1$ such that for every t with $0 \leq t \leq \delta$ we have $f_k(t) < \varepsilon$ for all k. Now, using Lemma 2.2, we get

$$\begin{aligned}&\left\{ r \in \mathbb{N} : h_r^{-1} \sum_{k \in I_r} \left[f_k \left(\| \Delta^m x_k - L, z_1, z_2, ..., z_{n-1} \| \right) \right]^{p_k} \geq \varepsilon \right\} \\ &= \left\{ r \in \mathbb{N} : h_r^{-1} \left(h_r . \max \left\{ \varepsilon^h, \varepsilon^H \right\} \right) \geq \varepsilon \right\} \\ &\cup \left\{ \begin{array}{c} r \in \mathbb{N} : h_r^{-1} \max \left\{ \left(2 f_k(1) \delta^{-1} \right)^h, \left(2 f_k(1) \delta^{-1} \right)^H \right\} \\ \times \sum_{k \in I_r} \left(\| \Delta^m x_k - L, z_1, z_2, ..., z_{n-1} \| \right)^{p_k} \end{array} \right\}.\end{aligned}$$

This completes the proof. The other case can be proved similarly. □

Theorem 2.4. *Let $\Omega = (f_k)$ be a sequence of moduli. If*

$$\lim_t \sup \frac{f_k(t)}{t} = \gamma > 0 \textit{ for all } k,$$

then

$$[N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]_0^I = [N_\theta, \Delta^m, p, \|., ..., .\|, X_s]_0^I$$

and

$$[N_\theta, \Delta^m, p, \|., ..., .\|, X_s]^I = [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I.$$

Proof. In Theorem 2.3, it was shown that

$$[N_\theta, \Delta^m, p, \|., ..., .\|, X_s]^I \subset [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I.$$

We must show that $[N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I \subset [N_\theta, \Delta^m, p, \|., ..., .\|, X_s]^I$. For any modulus function, the existence of positive limit given by with γ was given Maddox [10]. Now $\gamma > 0$ and let $x \in [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I$. Since $\gamma > 0$, for every $t > 0$ we write $f_k(t) \geq \gamma t$ for all k. From this inequality

$$\begin{aligned}&h_r^{-1} \sum_{k \in I_r} \left[f_k \left(\| \Delta^m x_k - L, z_1, z_2, ..., z_{n-1} \| \right) \right]^{p_k} \\ &\geq \gamma^H h_r^{-1} \sum_{k \in I_r} \left(\| \Delta^m x_k - L, z_1, z_2, ..., z_{n-1} \| \right)^{p_k}\end{aligned}$$

and this inequality gives the result. □

Corollary 2.5. *Let $\Omega_1 = (f_k)$ and $\Omega_2 = (g_k)$ be sequences of moduli. If*

$$\lim_t \sup \frac{f_k(t)}{g_k(t)} < \infty$$

implies

$$[N_\theta, \Omega_1, \Delta^m, p, \|., ..., .\|, X_s]_0^I \subset [N_\theta, \Omega_2, \Delta^m, p, \|., ..., .\|, X_s]_0^I$$

and

$$[N_\theta, \Omega_1, \Delta^m, p, \|., ..., .\|, X_s]^I \subset [N_\theta, \Omega_2, \Delta^m, p, \|., ..., .\|, X_s]^I.$$

Theorem 2.6. *Let $\left(X, \|., ..., .\|_{X_s} \right)$ and $\left(X, \|., ..., .\|_{X_E} \right)$ be standard and Euclid n-normed spaces, respectively. Then*

$$\begin{aligned}&[N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_s]^I \cap [N_\theta, \Omega, \Delta^m, p, \|., ..., .\|, X_E]^I \\ &\subset [N_\theta, \Omega, \Delta^m, p, (\|., ..., .\| X_s + \|., ..., .\| X_E)]^I.\end{aligned}$$

Proof. We have the following inclusion:

$$\left\{\begin{array}{c} r\in\mathbb{N}: h_r^{-1}\sum\limits_{k\in I_r}[f_k((\|.,...,.\|_{X_s}+\|.,...,.\|_{X_E}) \\ (\Delta^m x_k - L, z_1,...,z_{n-1}))]^{p_k}\geq\varepsilon \end{array}\right\}$$

$$\subset\left\{r\in\mathbb{N}: K.h_r^{-1}\sum_{k\in I_r}[f_k\left(\|\Delta^m x_k - L, z_1,...,z_{n-1}\|_{X_s}\right)]^{p_k}\geq\varepsilon\right\}$$

$$\cup\left\{r\in\mathbb{N}: K.h_r^{-1}\sum_{k\in I_r}[f_k\left(\|\Delta^m x_k - L, z_1,...,z_{n-1}\|_{X_E}\right)]^{p_k}\geq\varepsilon\right\}$$

by using (2.1). This completes the proof. □

Theorem 2.7. *Let $\Omega_1=(f_k)$ and $\Omega_2=(g_k)$ be sequences of moduli. Then*

(i)

$$[N_\theta,\Omega_1,\Delta^m,p,\|.,...,.\|,X_s]_0^I\subset[N_\theta,\Omega_1\circ\Omega_2,\Delta^m,p,\|.,...,.\|,X_s]_0^I$$

and

$$[N_\theta,\Omega_1,\Delta^m,p,\|.,...,.\|,X_s]^I\subset[N_\theta,\Omega_1\circ\Omega_2,\Delta^m,p,\|.,...,.\|,X_s]^I.$$

(ii)

$$[N_\theta,\Omega_1,\Delta^m,p,\|.,...,.\|,X_s]_0^I\cap[N_\theta,\Omega_2,\Delta^m,p,\|.,...,.\|,X_s]_0^I$$
$$\subset[N_\theta,\Omega_1+\Omega_2,\Delta^m,p,\|.,...,.\|,X_s]_0^I$$

and

$$[N_\theta,\Omega_1,\Delta^m,p,\|.,...,.\|,X_s]^I\cap[N_\theta,\Omega_2,\Delta^m,p,\|.,...,.\|,X_s]^I$$
$$\subset[N_\theta,\Omega_1+\Omega_2,\Delta^m,p,\|.,...,.\|,X_s]^I.$$

Proof. Let $(x_k)\in[N_\theta,\Omega_1,\Delta^m,p,\|.,...,.\|,X_s]^I$. Let $0<\varepsilon<1$ and δ with $0<\delta<1$ such that $f_k(t)<\varepsilon$ for $0<t<\delta$. Let $y_k=g_k(\|\Delta^m x_k - L, z_1,z_2,...,z_{n-1}\|)$. Let

$$h_r^{-1}\sum_{k\in I_r}[f_k(y_k)]^{p_k}=h_r^{-1}\sum_1[f_k(y_k)]^{p_k}+h_r^{-1}\sum_2[f_k(y_k)]^{p_k},$$

where the first summation is over $y_k\leq\delta$ and the second is over $y_k>\delta$. Then $h_r^{-1}\sum\limits_1[f_k(y_k)]^{p_k}\leq\varepsilon^H$ and for $y_k>\delta$, we use the fact that

$$y_k<\frac{y_k}{\delta}<1+\left[\left|\frac{y_k}{\delta}\right|\right],$$

where $[|z|]$ denotes the integer part of z. From the properties of modulus function, we have for $y_k>\delta$

$$f_k(y_k)<\left(1+\left[\left|\frac{y_k}{\delta}\right|\right]\right)f_k(1)\leq 2f_k(1)\frac{y_k}{\delta}$$

for all k. Hence

$$h_r^{-1}\sum_2[f_k(y_k)]^{p_k}\leq\left[2\frac{f_k(1)}{\delta}\right]^H h_r^{-1}\sum_2[y_k]^{p_k}$$

which together with $h_r^{-1}\sum\limits_1[f_k(y_k)]^{p_k}\leq\varepsilon^H$ yields

$$h_r^{-1}\sum_{k\in I_r}[f_k(y_k)]^{p_k}\leq\varepsilon^H+\max\left(1,(2f_k(1)\delta^{-1})^H\right)\sum_{k\in I_r}[f_k(y_k)]^{p_k}$$

and this completes the proof.

(ii) Let $(x_k) \in [N_\theta, \Omega_1, \Delta^m, p, \|.,...,.\|, X_s]^I \cap [N_\theta, \Omega_2, \Delta^m, p, \|.,...,.\|, X_s]^I$. The fact that

$$h_r^{-1} \sum_{k \in I_r} [(f_k + g_k)(\|\Delta^m x_k - L, z_1, z_2, ..., z_{n-1}\|)]^{p_k}$$
$$\leq K.h_r^{-1} \sum_{k \in I_r} [f_k(\|\Delta^m x_k - L, z_1, z_2, ..., z_{n-1}\|)]^{p_k}$$
$$+K.h_r^{-1} \sum_{k \in I_r} [g_k(\|\Delta^m x_k - L, z_1, z_2, ..., z_{n-1}\|)]^{p_k}$$

gives the result. □

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