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ON NON-TRIANGULAR METRIC AND JS-METRIC SPACES AND RELATED CONSEQUENCES VIA $\widehat{Man}(\mathbb{R})-$CONTRACTIONS

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Abstract. In this work, we establish some new fixed point theorems in the set of non-triangular metric spaces as a new and real generalization of JS-metric spaces. Moreover, we show that many of known or unknown fixed point results in such spaces can conclude from $\widehat{Man}(\mathbb{R})-$contractions.

1. Introduction

In [1] Banach opened up a new way in nonlinear analysis, upon which, various applications in a variety of sciences have appeared. After this interesting principle, several authors generalized this principle by introducing the various contractions on metric spaces (see, e.g., [2, 15, 16, 6, 10, 14]).

In 2014, the notion of *manageable function* introduced by Du and Khojasteh [12, 11] to generalize and unify the several existing fixed point results in the literature which continued by Karapinar and Khojasteh [17] and Shahzad et al. [9].

After that, Jleli and Samet [7] introduced a generalization of metric spaces that recovers a large class of topological spaces including standard metric spaces, b-metric spaces, dislocated metric spaces and modular spaces called $JS-$metric spaces. Later, this new concept studied by many authors and worthwhile results in this direction obtained (see [3, 4, 5] for more details).

1.1. **Highlight:** At a glance, the following highlight can help readers to view the results of manuscript and quickly identify the main aims without to dig through it.

- In this research, firstly, we introduce non-triangular metric space and create an example to demonstrate that non-triangular metric space is a real generalization of $JS-$metric space. Finally, we present some new fixed point theorems to convey our main results.

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2. $JS-$metric space

In this section, we recall $JS-$metric spaces and some related properties. From now on, we reserve the symbols $\mathbb{R}, \mathbb{R}^+, \mathbb{N}$ for the reals, positive reals and natural number, respectively.

Let X be a nonempty set and let $\varrho : X \times X \to [0, +\infty)$ be a given mapping. For every $x \in X$, define the sets:

$$\mathcal{M}(\varrho, X, x) = \{\{x_n\} \subset X : \lim_{n\to\infty} \varrho(x_n, x) = 0\}.$$

Definition 2.1. *We say that ϱ is a $JS-$metric on X if it satisfies the following conditions:*

(a_1) *for each pair $(x, y) \in X \times X$, we have*

$$\varrho(x, y) = 0 \quad \textit{implies} \quad x = y,$$

(a_2) *for each pair $(x, y) \in X \times X$, we have*

$$\varrho(x, y) = \varrho(y, x),$$

(a_3) *there exists $\kappa \geq 1$ such that for all $x, y \in X$, if $\{x_n\} \in \mathcal{M}(X, \varrho, x)$*

$$\varrho(x, y) \leq \kappa \limsup_{n\to\infty} \varrho(x_n, y).$$

In this case, we say the pair (X, ϱ) is a $JS-$metric space by modulus κ.

Definition 2.2. *Let (X, ϱ) be an $JS-$metric space.*

(b_1) *We say that $\{x_n\}$ $\varrho-$converges to x if $\{x_n\} \in \mathcal{M}(\varrho, X, x)$,*
(b_2) *if $\{x_n\}$ $\varrho-$converges to x and $\varrho-$converges to y then $x = y$,*
(b_3) *$\{x_n\}$ is $\varrho-$Cauchy sequence if $\lim_{m,n\to\infty} \varrho(x_n, x_{n+m}) = 0$,*
(b_4) *(X, ϱ) is said to be $\varrho-$complete if every Cauchy sequence in X is convergent to some element in X.*

3. Non-Triangular Metric Space

Very recently, Khojasteh and Khandani [8] introduced non-triangular metric space and obtain some fixed point results which are the generalization of some new recent results in the literature.

Definition 3.1. [8] *Let X is a nonempty set and let $\rho : X \times X \to \mathbb{R}^+$ is a mapping. We say that ρ is a non-triangular metric on X if it satisfies the following conditions:*

(c_1) *for each pair $x \in X$, we have $\rho(x, x) = 0$,*
(c_2) *for each $x, y \in X$, and $\{x_n\} \subset X$ such that $\lim_{n\to\infty} \rho(x_n, x) = 0$, and $\lim_{n\to\infty} \rho(x_n, y) = 0$, then $x = y$.*

Nota that if $x, y \in X$ and $\rho(x, y) = 0$ then taking $x_n = x$ and applying (a_1) and (a_2) we have $x = y$.

Definition 3.2. [8] *Let (X, ρ) be a non-triangular metric space.*

(d_1) *We say that $\{x_n\}$ $\rho-$converges to x if $\lim_{n\to\infty} \rho(x_n, x) = 0$*
(d_2) *$\{x_n\}$ is $\rho-$Cauchy sequence if $\lim_{n\to\infty} \sup\{\rho(x_n, x_m) : m \geq n\} = 0$,*
(d_3) *(X, ρ) is said to be $\rho-$complete if every Cauchy sequence in X is convergent to some element in X.*

The following example shows that non-triangular metric space is a real generalization of generalized metric space in a sense of Samet's concept.

Example 3.3. *Let $X = [0, +\infty)$ endowed with Euclidean metric. Define*

$$\begin{cases} \rho(x,y) = \frac{\sqrt{x+y}}{\sqrt{x+y}+1} & if\ x \neq 0\ and\ y \neq 0 \\ \rho(0,x) = \rho(x,0) = \frac{x}{2} & for\ all\ x \in X \end{cases}$$

Conditions (c_1) is trivially satisfied. We need to verify condition (c_2). If $x, y \in X$ and $\{x_n\}$ be a sequence in X such that $\rho(x_n, x) \to 0$ and $\rho(x_n, y) \to 0$. Note that the only convergent sequences in X are ones which are convergent to zero, because, suppose that $\{x_n\}$ be convergent to x. Then,

$$0 = \lim_{n\to\infty} \rho(x_n, x) = \lim_{n\to\infty} \frac{\sqrt{x_n + x}}{\sqrt{x_n + x} + 1},$$

and this holds, if and only if, $x_n \to 0$ in $\mathbb{R}$, and $x = 0$. Thus, (a_3) trivially holds.

For all $n \in \mathbb{N}$ and for each $y \in X$,

$$\rho(x_n, y) = \begin{cases} \frac{\sqrt{x_n+y}}{\sqrt{x_n+y}+1} & if\ x_n \neq 0, \\ \frac{y}{2} & if\ x_n = 0. \end{cases}$$

Since $\{x_n\}$ is a convergent sequence to zero. If there exists $C \geq 1$ such that

$$\frac{y}{2} = \rho(0,y) \leq C \limsup_{n\to\infty} \rho(x_n, y) = C \limsup_{n\to\infty} \frac{\sqrt{x_n + y}}{\sqrt{x_n + y} + 1} = C \frac{\sqrt{y}}{\sqrt{y} + 1},$$

we have $C \geq \frac{y\sqrt{y}+y}{2\sqrt{y}} \geq \frac{y}{2}$. Therefore, there is no bound for C, by which,

$$\rho(y, 0) \leq C \limsup_{n\to\infty} \rho(y, x_n).$$

4. Main results

In current section, combining the concept of non-triangular metric space and manageable functions, we present new and generalized fixed point results by which we can obtain some unknown fixed point theorems.

Definition 4.1. *A* manageable function *is a mapping $\upsilon : [0, \infty) \times [0, \infty) \to \mathbb{R}$ satisfies the following conditions:*

- (υ_1)**:** *$\upsilon(t, s) < s - t$ for all $t, s > 0$,*
- (υ_2)**:** *for all sequences $\{t_n\}_n, \{s_n\}_n \subseteq (0, +\infty)$, if $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n$ then*

$$\limsup_{n\to\infty} \frac{t_n + \upsilon(t_n, s_n)}{s_n} < 1. \tag{4.1}$$

Denote $\widehat{Man}(\kappa, \mathbb{R})$ the family of all manageable functions $\upsilon : [0, \infty) \times [0, \infty) \to \mathbb{R}$.

Definition 4.2. *Let (X, ϱ) be an non-triangular metric space, $T : X \to X$ a mapping, and $\upsilon \in \widehat{Man}(\kappa, \mathbb{R})$. Then, T is called a $\widehat{Man}(\mathbb{R})$-contraction with respect to υ if the following condition holds*

$$\upsilon(\varrho(Tx, Ty), \varrho(x, y)) \geq 0 \quad for\ all \quad x, y \in X. \tag{4.2}$$

Remark. *Considering* (υ_1) *of Definition 4.1, if* T *is a* $\widehat{Man}(\mathbb{R})$*-contraction with respect to* $\upsilon \in \widehat{Man}(\kappa, \mathbb{R})$*, then*

$$\varrho(Tx, Ty) < \varrho(x, y) \text{ for all distinct } x, y \in X.$$

In next theorem, we prove the existence and uniqueness the fixed point of a $\widehat{Man}(\mathbb{R})$-contraction.

For every $x_0 \in X$, let

$$\delta(\varrho, T, x_0) = \sup\{\varrho(T^i(x_0), T^j(x_0) : i, j \in \mathbb{N})\}.$$

Theorem 4.3. *Let* (X, ϱ) *be a* $\varrho-$*complete non-triangularmetric space and* $T : X \to X$ *be a* $\widehat{Man}(\mathbb{R})$*-contraction with respect to* υ*. If* $\delta(\varrho, T, x_0) < \infty$ *then,* T *has a unique fixed point* w *in* X*.*

Proof. Let $x_0 \in X$ such that $\delta(\varrho, T, x_0) < \infty$. Let $x_0 \in X$ is arbitrary and let $x_1 = Tx_0$. If $x_1 = x_0$ then we have nothing to prove. Thus, suppose that $x_1 \neq x_0$ and for every $n \in \mathbb{N}$, define $x_{n+1} = Tx_n$.

Suppose that $A = \{(\varrho(Tx, Ty), \varrho(x, y)) \in (0, +\infty) \times (0, +\infty) : x, y \in X\}$ and let us define

$$\alpha(t, s) = \begin{cases} \frac{t+\upsilon(t,s)}{s} & (t, s) \in A \\ 0 & Otherwise \end{cases}$$

Note that if $\upsilon(t, s) \geq 0$, for all $t, s \in (0, +\infty)$, then

$$t \leq \alpha(t, s)s,$$

and also $0 < \alpha(t, s) < 1$, for all $(t, s) \in A$.

Since T is $\widehat{Man}(\mathbb{R})-$contraction, we have

$$\varrho(Tx, Ty) \leq \alpha(\varrho(Tx, Ty), \varrho(x, y))\varrho(x, y), \tag{4.3}$$

for all $x, y \in X$. We divide the rest of proof into two cases

Case 1: The sequence $\{x_n\}$ is Cauchy. Let

$$\delta_n = \sup\{\varrho(T^i(x_0), T^j(x_0)) : i, j \geq n\}.$$

Note that $0 \leq \delta_{n+1} \leq \delta_n$ and $\delta_1 = \delta(\varrho, T, x_0) < \infty$. Thus, $\delta_n < \infty$ for all $n \in \mathbb{N}$. Therefore, $\{\delta_n\}$ is monotonic bounded sequence and so is convergent. Thus, there exists $\delta \geq 0$ such that $\lim_{n\to\infty} \delta_n = \delta$. We shall show that $\delta = 0$. If $\delta > 0$, then by the definition of δ_n, for every $k \in \mathbb{N}$ there exists n_k, m_k such that $m_k > n_k \geq k$ and

$$\delta_k - \frac{1}{k} < \varrho(T^{m_k}(x_0), T^{n_k}(x_0)) \leq \delta_k.$$

Hence

$$\lim_{k\to\infty} \varrho(x_{m_k-1}, x_{n_k-1}) = \lim_{k\to\infty} \varrho(T^{m_k}(x_0), T^{n_k}(x_0)) = \delta. \tag{4.4}$$

From $m_k - 1 > n_k - 1 \geq k - 1$, we have $\varrho(T^{m_k-1}(x_0), T^{n_k-1}(x_0)) \leq \delta_{k-1}$ and the fact that T is $\widehat{Man}(\mathbb{R})-$contraction, we conclude

$$\varrho(T^{m_k}(x_0), T^{n_k}(x_0)) \leq \varrho(x_{m_k-1}, x_{n_k-1}) \leq \delta_{k-1}. \tag{4.5}$$

Taking limit from both side of (4.5) and considering (4.4), one can conclude that

$$\lim_{k\to\infty} \varrho(x_{m_k-1}, x_{n_k-1}) = \delta. \tag{4.6}$$

Since T is a $\widehat{Man}(\mathbb{R})$-contraction with respect to υ,

$$\begin{aligned}\varrho(x_{m_k}, x_{n_k}) &\leq \alpha(\varrho(x_{m_k}, x_{n_k}), \varrho(x_{m_k-1}, x_{n_k-1})) \\ &\quad \times \varrho(x_{m_k-1}, x_{n_k-1}).\end{aligned} \tag{4.7}$$

Using (4.4), (4.6) and ($\upsilon 2$) and taking limit on both side of (4.7), we have

$$\delta \leq \limsup_{k\to\infty} \alpha(\varrho(x_{m_k}, x_{n_k}), \varrho(x_{m_k-1}, x_{n_k-1})) < \delta \tag{4.8}$$

which is a contradiction. This contradiction proves that $\delta = 0$ and so $\{x_n\}$ is a $\varrho-$Cauchy sequence. Since (X, ϱ) is $\varrho-$complete, there exists some $\omega \in X$ such that $\{x_n\}$ is $\varrho-$convergent to ω.

Case 2: w is the unique fixed point of T.

Since T is a $\widehat{Man}(\mathbb{R})-$contraction, for all $n \geq n_0$, we have

$$\varrho(x_{n+1}, T(\omega)) \leq \alpha(\varrho(x_{n+1}, T(\omega)), \varrho(x_n, \omega))\varrho(x_n, \omega) < \varrho(x_n, \omega), \tag{4.9}$$

and by taking limit from (4.9), we have $\{x_n\}$ is $\varrho-$ converges to $T(\omega)$. By the uniqueness of the limit (see Definition 2.2), we get $\omega = T(\omega)$; that is, ω is a fixed point of T. Also, the uniqueness of the fixed point is easily verified by the contraction.

□

Example 4.4. *Let $X = [0, 1]$ and let $\rho : X \times X \to [0, +\infty]$, defined as follows*

$$\begin{cases} \rho(0, x) = \rho(x, 0) = \frac{x}{2} & x \in X, \\ \rho(x, y) = x + y & x \neq 0, y \neq 0. \end{cases}$$

Let $Tx = \frac{x}{x+2}$, for all $x \in X$. Then, we have

$$\rho(T(x), T(0)) = \frac{x}{2(x+2)}$$

and for each $x, y \in X - \{0\}$, we have

$$\rho(T(x), T(y)) = \frac{x}{2(x+2)} + \frac{y}{2(y+2)}$$

(X, ρ) is a $JS-$metric space that is not a standard metric since the triangle inequality does not hold: If $x, y \in X - \{0\}$, then we have $\rho(x, y) = x + y$ and $\rho(x, 0) + \rho(0, y) = \frac{x+y}{2}$, and thus

$$\rho(x, y) > \rho(x, 0) + \rho(0, y),$$

Moreover, X is $\rho-$complete non-triangular space and one can easily check that T is a $\widehat{Man}(\mathbb{R})$-contraction with $\upsilon(t, s) = \frac{1}{6}s - t$ and also it has a fixed point.

Here we present a characterization for $\widehat{Man}(\mathbb{R})$-contractions as follows:

Theorem 4.5. *Let (X, ρ) be a non-triangular metric space with the modulus $\kappa \geq 1$ and let $T : X \to X$ be an operator. Then, T is a $\widehat{Man}(\mathbb{R})$-contraction if and only if T satisfies the following contraction*

$$\rho(Tx, Ty) \leq \alpha(\rho(Tx, Ty), \rho(x, y))\rho(x, y), \tag{4.10}$$

for all $x, y \in X$, where $\alpha : R_+ \times R_+ \to R_+$ is a function such that $0 \leq \alpha(r, s) < 1$, for all $r, s \geq 0$ and for all two bounded sequences $\{r_n\}_n, \{s_n\}_n \subseteq (0, +\infty)$, if $\lim_{n\to\infty} r_n = \lim_{n\to\infty} s_n = L$, then

$$\limsup_{n\to\infty} \alpha(r_n, s_n) < 1. \tag{4.11}$$

Proof. Let T be any $\widehat{Man}(\mathbb{R})$-contraction on X. Define

$$\alpha(t,s)=\begin{cases}\frac{t+\upsilon(t,s)}{s} & (t,s)\in A\\ 0 & Otherwise\end{cases}$$

Analogous the proof of Theorem 4.3, we see that $0<\alpha(t,s)<1$, for all $t,s\in A$, in which $A=\{(\rho(Tx,Ty),\rho(x,y))\in(0,+\infty)\times(0,+\infty):x,y\in X\}$. We deduce that

$$\rho(Tx,Ty)\leq\alpha(\rho(Tx,Ty),\rho(x,y))\rho(x,y), \tag{4.12}$$

for all $x,y\in X$.

Now assume T be an operator satisfies in (4.10), for some α as given in the assumptions. Define $\upsilon(r,s)=\alpha(r,s)s-r$. It is easy to check that

- $\upsilon(r,s)=\alpha(r,s)s-r<s-r$,
- for all two bounded sequences $\{r_n\}_n,\{s_n\}_n\subseteq(0,+\infty)$, if $\lim_{n\to\infty}r_n=\lim_{n\to\infty}s_n=L\geq 0$, then

$$\limsup_{n\to\infty}\alpha(r_n,s_n)<1. \tag{4.13}$$

Therefore, υ is a $\kappa-$manageable function. Now, we show that T is a $\widehat{Man}(\mathbb{R})$-contraction. By the definition of T, we have

$$\upsilon(\rho(T(x),T(y)),\rho(x,y))=\alpha(\rho(T(x),T(y)),\rho(x,y))\rho(x,y)-\rho(T(x),T(y))\geq 0 \tag{4.14}$$

and this completes the proof. □

5. Consequences

In the following, we obtain some new and known results via manageable functions, by which, one can obtain known fixed point theorems to work on other aspects of these results like best proximity points, multi valued mappings and other applications related to solve the nonlinear optimization problems.

Corollary 5.1. *Let (X,ρ) be a $\rho-$complete non-triangular metric space and let $T\colon X\to X$ is a mapping satisfies in the following condition:*

$$\rho(Tx,Ty)\leq\lambda\rho(x,y),$$

for all $x,y\in X$, where $0\leq\lambda<1$. If $\delta_1(x_0)<\infty$ for some $x_0\in X$, then T has a unique fixed point in X.

Proof. Define $\upsilon_B\colon[0,\infty)\times[0,\infty)\to\mathbb{R}$ by

$$\upsilon_B(t,s)=\lambda s-t \quad\text{for all}\quad s,t\in[0,\infty).$$

Note that, the mapping T is a $\widehat{Man}(\mathbb{R})$-contraction with respect to $\upsilon_B\in\widehat{Man}(\mathbb{R})$. Therefore, the result follows by taking $\upsilon=\upsilon_B$ in Theorem 4.3. □

Corollary 5.2. *Let (X,ρ) be a $\rho-$complete non-triangular metric space and let $T\colon X\to X$ be a mapping satisfies in the following condition:*

$$\rho(Tx,Ty)\leq\rho(x,y)-\Im(\rho(x,y)),$$

for all $x,y\in X$, where $\Im\colon[0,\infty)\to[0,\infty)$ is a mapping such that

$$\liminf_{t\to s^+}\frac{\Im(t)}{t}>0,$$

for all $0 < s$. *If* $\delta(\varrho, T, x_0) < \infty$ *for some* $x_0 \in X$, *then* T *has a unique fixed point in* X.

Proof. Define $\upsilon_R\colon [0,\infty) \times [0,\infty) \to \mathbb{R}$ by

$$\upsilon_R(t,s) = s - \Im(s) - t,$$

for all $s,t \in [0,\infty)$. Note that, the mapping T is a $\widehat{Man}(\mathbb{R})$-contraction with respect to $\upsilon_R \in \widehat{Man}(\mathbb{R})$. Therefore, the result follows by taking $\upsilon = \upsilon_R$ in Theorem 4.3. $\square$

Corollary 5.3. *Let* (X,ρ) *be a* $\rho-$*complete non-triangular metric space and let* $T : X \to X$ *is a mapping. Suppose that for every* $x, y \in X$,

$$\rho(Tx,Ty) \leq \Im(\rho(x,y))\rho(x,y),$$

for all $x, y \in X$, *where* $\Im : [0,+\infty) \to [0,1)$ *is a mapping such that* $\limsup_{t\to\ s^+} \Im(t) < 1$, *for all* $0 < s$. *If* $\delta(\varrho, T, x_0) < \infty$ *for some* $x_0 \in X$, *then* T *has a unique fixed point.*

Proof. Define $\upsilon_T\colon [0,\infty) \times [0,\infty) \to \mathbb{R}$ by

$$\upsilon_T(t,s) = s\Im(s) - t,$$

for all $s,t \in [0,\infty)$. Note that, the mapping T is a $\widehat{Man}(\mathbb{R})$-contraction with respect to $\upsilon_T \in \widehat{Man}(\mathbb{R})$. Therefore, the result follows by taking $\upsilon = \upsilon_T$ in Theorem 4.3. $\square$

Corollary 5.4. *Let* (X,ρ) *be a* $\rho-$*complete non-triangular metric space and let* $T : X \to X$ *is a mapping. Suppose that for every* $x, y \in X$,

$$\rho(Tx,Ty) \leq \Im(\rho(x,y)),$$

for all $x, y \in X$, *where* $\Im : [0,+\infty) \to [0,+\infty)$ *is a mapping such that* $\Im(t) < t$ *for all* $t > 0$, $\Im(0) = 0$ *and*

$$\limsup_{t\to s^+} \frac{\Im(t)}{t} < 1$$

for all $0 < s$. *If* $\delta(\varrho, T, x_0) < \infty$ *for some* $x_0 \in X$, *then* T *has a unique fixed point.*

Proof. Define $\upsilon_{BW}\colon [0,\infty)\times[0,\infty) \to \mathbb{R}$ by $\upsilon_{BW}(t,s) = \Im(s) - t$ for all $t,s \in [0,\infty)$. Note that, the mapping T is a $\widehat{Man}(\mathbb{R})$-contraction with respect to $\upsilon_{BW} \in \widehat{Man}(\mathbb{R})$. Therefore, the result follows by taking $\upsilon = \upsilon_{BW}$ in Theorem 4.3. $\square$

Overall summery and future directions

Broadly translated our findings indicate that most of known fixed point achievements, by taking into account triangle inequality or other its generalizations, have analogous perception in which some struggle have been accomplished to obtain desired result. However, omitting the triangle inequality makes the results more difficult to prove, rehearsing fixed point results without considering triangle inequality makes them worthwhile because, most of the spaces in natural phenomenon, in the most well-behaved conditions, suffer from lacking triangle inequality.

In this research, after introducing non-triangular metric space and its comparing with JS-metric space, we presented an instance to show that triangular metric space is a real generalization of JS-metric space. Also, we present a characterization of $\widehat{Man}(\mathbb{R})-$contractions and show that this kind of contraction has also a normal structure like previous ones. Finally, we terminated our results by proving the existence a fixed point for some new results in non-triangular metric spaces.

Further studies need to be carried out in order to study on multi valued version of fixed point theorems in non-triangular metric spaces by studying non-triangular Hausdorff metric space. It is worth mentioning that applying non-triangular Hausdorff metric space to obtain new results needs more struggle to prove and is as hard as one expect. Also, we think that working on well-posed problems and finding appropriate conditions on multi valued contractions in order to find strict fixed point (endpoint) will give rise open questions and high attention with respect to those results. We refer authors to [10, 13] for more details.

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