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ON SOME INEQUALITIES AND $\sigma_{B(r,s)}$-CONSERVATIVE MATRICES

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Abstract. Let V_σ denote the spaces of σ-convergent sequences introduced by P. Schaefer [Infinite matrices and invariant means, Proc. Amer. Math. Soc. 36(1972), 104-110] and $\widehat{V_\sigma}$ also be the domain of the generalized difference matrix $B(r,s)$ in the sequences spaces V_σ. In this paper, we have determined a class of $\sigma_{B(r,s)}$-conservative matrices studying some inequalities which are analogous to Knopp' s Core Theorem.

1. Introduction

Let w be the spaces of all real or complex valued sequences. Then, each linear subspace of w is called a sequence space. We write ℓ_∞, c, c_0 and ℓ_1, cs, and bs are used for the sequence spaces of all bounded , convergent, and null sequences, absolutely convergent series, convergent series, and bounded series, respectively. Let λ and μ be two sequence spaces and $A = (a_{nk})$ an infinite matrix of real or complex numbers a_{nk}, where $n, k \in \mathbb{N} = \{0, 1, 2, ...\}$. Then, A defines a matrix mapping from λ to μ by $A : \lambda \longrightarrow \mu$ if for every sequence $x = (x_k) \in \lambda$ the sequence $Ax = \{(Ax)_n\}$, the A-transform of x, is in μ where

$$(Ax)_n = \sum_k a_{nk}x_k, \ (n \in \mathbb{N}). \tag{1.1}$$

By $(\lambda : \mu)$, we denote the class of matrices A such that $A : \lambda \longrightarrow \mu$. Thus , $A \in (\lambda : \mu)$ if and only if the above series converges for each $n \in \mathbb{N}$ and every $x \in \lambda$, and we have $Ax = \{(Ax)_n\}_{n\in\mathbb{N}} \in \mu$ for all $x \in \lambda$. The matrix domain λ_A of an infinite matrix A in a sequence spaces λ is defined by $\lambda_A = \{x = x_k \in \omega : Ax \in \lambda\}$. If we take $\lambda = c$, then c_A is called convergence domain of A. We write the limit of Ax as $\lim_A x = \lim_{n\to\infty} \sum_k a_{nk}x_k$, and A is called regular if $\lim_A x = \lim x$ for each convergent sequence x. By using the matrix domain of a particular limitation method so many sequence spaces have been built, we can see ([2], [3], [4], [5], [6], [7], [9], [14], [15], [16], [17], [20], [21], [23], [24], [25], [26], [27], [29], [31], [35], [36]) . Finally, the new technique for deducing certain topological properties, such as AB-, KB-, AD-,properties, solidity and monotonicity etc., and determining the α-, β-,

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and γ- duals of the domain of a triangle matrix in a sequence space was given by Altay and Başar [1]. Furthermore, quite recently, Başar and Kirişci [10] introduced the new sequence spaces $\hat{f}$ derived from the space f of almost convergent sequences by means of the domain of the generalized difference matrix $B(r,s)$.

Let σ be a one-to-one mapping from $\mathbb{N}$ into itself and $T : \ell_\infty \longrightarrow \ell_\infty$ a linear operator defined by $Tx = (Tx_k) = (x_{\sigma(k)})$. An element $\varphi \in \ell_\infty^{'}$, the conjugate space of ℓ_∞, is called an invariant mean or a σ-mean if and only if (i) $\phi(x) \geq 0$ when the sequence $x = (x_k)$ has $x_k \geq 0$ for all $k \in \mathbb{N}$, (ii) $\varphi(e) = 1$, where $e = (1, 1, 1, ...)$ and (iii) $\phi(Tx) = \varphi(x)$ for all $x \in \ell_\infty$. A sublinear functional P on ℓ_∞ is said to generate σ-means if $\phi \in \ell_\infty^{'}$ and $\varphi \leq P \Rightarrow \varphi$ is a σ-mean, to dominate σ means if $\varphi \leq P$ for all $\varphi \in M$ where $\varphi \leq P$ means that $\varphi(x) \leq P(x)$ for all $x \in \ell_\infty$. It is shown [32] that the sublinear functional $V(x) = \sup_n \limsup_p t_{pn}(x)$ both generate and dominates σ-means where

$$t_{pn}(x) = (x_n + Tx_n + ... + T^p x_n)/(p+1), \; t_{-1,n}(x) = 0. \tag{1.2}$$

A bounded sequence x is called σ- convergent to s if $V(x) = -V(-x) = s$ and it can be shown [34] that

$$V_\sigma = \{x \in \ell_\infty : \lim_p t_{pn}(x) = s \text{ uniformly in } n, \text{ s} = \sigma - \lim x\}. \tag{1.3}$$

In this case we write $\sigma - \lim x = s$.

Throughout this paper we consider the mapping σ such that $\sigma^p(k) \neq k$ for all positive integers $k \geq 0$ and $p \geq 1$, where $\sigma^p(k)$ is the pth iterate of σ at k. Thus, a σ-mean extends the limit functional on c in the sense that $\varphi(x) = \lim x$ for all $x \in c$ (see [22]). Consequently, $c \subset V_\sigma$ where V_σ is the set of bounded sequences all of whose σ-means are equal.

In case $\sigma(k) = k+1$, a σ-mean is often called a Banach limit and V_σ is the set of almost convergent sequences, introduced by Lorentz (see [12]).

We say that a bounded sequence $x = (x_k)$ is σ-convergent if $x \in V_\sigma$. By Z, we denote the set of σ-convergent sequences with σ-limit zero. It is well known [33] that $x \in \ell_\infty$ if and only if $(Tx - x) \in Z$. $B(r,s) = \{b_{nk}(r,s)\}$ and $B(\widetilde{r}, \widetilde{s}) = \{b_{nk}(r_n, s_n)\}$ are the generalized difference matrix and double sequential band matrix, respectively defined by

$$b_{nk}(r,s) = \begin{cases} r, & k = n \\ s, & k = n-1 \\ 0, & \text{otherwise} \end{cases} \quad b_{nk}(\widetilde{r}, \widetilde{s}) = \begin{cases} r_n, & k = n \\ s_n, & k = n-1 \\ 0, & \text{otherwise} \end{cases}$$

for all $k, n \in \mathbb{N}$, where $r, s \in \mathbb{R} \setminus \{0\}$ and $\tilde{r} = (r_n)_{n=0}^{\infty}$ and $\tilde{s} = (s_n)_{n=0}^{\infty}$ be given convergent sequences of positive real numbers.

Now we introduce the new sequence spaces $\widehat{V_\sigma}$ as the set of all sequences whose $B(r,s)$-transforms are in the spaces V_σ, that is

$$\widehat{V_\sigma} = \left\{ x = (x_k) \in w : \exists \alpha \in \mathbb{C} \ni \lim_{m\to\infty} \sum_{j=0}^{m} \frac{s x_{\sigma^j(k)-1} + s x_{\sigma^j(k)}}{m+1} = \alpha \text{ uniformly in k} \right\}. \tag{1.4}$$

With the notation of (1.4), we can redefine the spaces $\widehat{V_\sigma}$ by

$$\widehat{V_\sigma} := V_{B(r,s)}.$$

Define the sequence $y = (y_k)$ by the $B(r,s)$-transform of a sequence $x = (x_k)$, i. e.

$$y_k = sx_{k-1} + rx_k, \ (k \in \mathbb{N}) \tag{1.5}$$

If we take $\sigma(k) = k+1$ in the (1.4) then $\widehat{V_\sigma}$ space is reduced to the space $\widehat{f}$ (see [10]).

We will call the matrices $A \in (c,c)$, $A \in (c, V_\sigma)$ and $A \in (c, S \cap \ell_\infty)$ conservative, σ-conservative and statistical (st)-conservative matrices. It is known [31] that A is conservative if and only if $||A|| = \sup_n \sum_k |a_{nk}| < \infty$, $a_k = \lim_n a_{nk}$ for each k, and $a = \lim_n \sum_k a_{nk}$. If A conservative, the number $\chi = \chi(A) = a - \sum_k a_k$ called the characteristic of A is of importance in summability.

Schaefer [34] has proved that A is σ-conservative if and only if $||A|| < \infty$, $\alpha_k = \sigma - \lim_n a_{nk}$ for each k, and $\alpha = \sigma - \lim \sum_k a_{nk}$.

Kolk [8] has shown that a matrix A is st -conservative if and only if $||A|| < \infty$, $t_k = st - \lim_n a_{nk}$ for each k, and $t = st - \lim \sum_k a_{nk}$.

In the case A is σ-conservative or st-conservative, similarly, we can define that is $\chi_\sigma = \chi_\sigma(A) = \alpha - \sum \alpha_k$ or $\chi_{st} = \chi_{st}(A) = t - \sum t_k$. If $\chi_\sigma \neq 0$, A is σ-coregular; otherwise, it is σ-conull. For any real λ we write $\lambda^+ = \max\{0, \lambda\}$, $\lambda^- = \max\{-\lambda, 0\}$. Then $\lambda = \lambda^+ + \lambda^-$ and $| \lambda | = \lambda^+ - \lambda^-$.

In what follows and throughout the paper we shall consider $(c, \widehat{V_\sigma})$ matrices class . $A \in (c, \widehat{V_\sigma})$ is said to be $\sigma_{B(r,s)}$-conservative if and only if

$$||A|| = \sup_n \sum_{k=0}^{\infty} |sa_{n-1,k} + ra_{nk}| < \infty$$

$$\lim_m \frac{1}{m+1} \sum_{j=0}^{m} sa_{\sigma^j(n)-1,k} + ra_{\sigma^j(n),k} = \alpha_k \text{ uniformly in } n, \text{ for each } k,$$

$$\lim_m \sum_{k=0}^{\infty} \frac{1}{m+1} \sum_{j=0}^{m} sa_{\sigma^j(n)-1,k} + ra_{\sigma^j(n),k} = \alpha \text{ uniformly in } n .$$

If the above conditions hold, then $\sigma_{B(r,s)} - \lim A_n(x) = \sum_k \alpha_k x_k + \ell(\alpha - \sum_k \alpha_k)$ for all $x \in c$.

Note that in the case A is $\sigma_{B(r,s)}$-conservative, the number $\chi_{B(r,s)} = \chi_{B(r,s)}(A) = \alpha - \sum_k \alpha_k$ is defined and it is said to be characteristic of A.

Let K be a subset of $\mathbb{N}$, the set of positive integers. Natural density δ of K is defined by

$$\delta(K) = \lim_n \frac{1}{n} |k \leq n : k \in K|,$$

where the vertical bars indicate the number of elements in the enclosed set. The number sequence $x = (x_k)$ is said to be statistically convergent to the number ℓ if for every ε, $\delta\{k : |x_k - \ell| \geq \varepsilon\} = 0$, [13]. In this case, we write $st - \lim x = \ell$. We shall also write st and st_0 to denote the sets of all statistically convergent sequences and statistically convergent to zero sequences. Fridy and Orhan [28], Lie and Fridy [30] have introduced the notions of the statistically boundedness, statistical-limit superior (st-limsup) and inferior (st-liminf), and also determined necessary and sufficient conditions for a matrix A to yield $L(Ax) \leq \beta(x)$ and $\beta(Ax) \leq \beta(x)$ for all $x \in \ell_\infty$. Das [11] has characterized a class of conservative matrices in terms of inequalities involving sublinear functionals on ℓ_∞. In [32] and [18, 19], the inequalities $q_\sigma(Ax) \leq L(x)$, $q_\sigma(Ax) \leq q_\sigma(x)$, $q_{\sigma^{(A)}}(Bx) \leq L(x)$, $q_{\sigma^{(A)}}(Bx) \leq q_\sigma(x)$, and $q_{\sigma^{(A)}}(Bx) \leq \beta(x)$, for all $x \in \ell_\infty$, have been studied.

Throughout this paper, we shall deal with the following sublinear functionals defined on $x \in \ell_\infty$:

$$L(x) = \limsup x, \qquad w(x) = \inf_{z \in Z_\sigma} L(x+z)$$

$$\alpha(x) = st - \liminf x, \qquad \beta(x) = st - \limsup x$$

where $t_{pn}(x)$ is defined as in (1.3). It is known that $V(x) = W(x)$ on ℓ_∞. In [11] Das characterized a class of conservative matrices in terms of inequalities involving sublinear functionals on ℓ_∞. In this paper, using the same technique, we have established some inequalities by using a class of $\sigma_{B(r,s)}$-conservative matrices, which are analogous to Knopp' s Core Theorem.

Firstly, we may list some lemmas which will be useful to our proofs.

Lemma 1.1 ([11],Th.1 (c)). *Let $\mathcal{A} = (a_{nk}(i))$ be conservative. Then, for some constant $\lambda \geq |\chi|$ and for all $x \in \ell_\infty$,*

$$\limsup_n \sup_i \sum_k (a_{nk}(i) - a_k)x_k \leq \frac{\lambda + \chi}{2} L(x) - \frac{\lambda - \chi}{2} \ell(x)$$

if and only if

$$\limsup_n \sup_i \sum_k |a_{nk}(i) - a_k| \leq \lambda \tag{1.6}$$

where χ is the characteristic of $\mathcal{A}$.

Lemma 1.2 ([11], Lemma 1). *Let $\mathcal{A} = (a_{nk}(i))$ be conservative and $\lambda \geq 0$. Then (1.6) holds if and only if*

$$\limsup_n \sup_i \sum_k (a_{nk}(i) - a_k)^+ \leq \frac{\lambda + \chi}{2}$$

and

$$\limsup_n \sup_i \sum_k (a_{nk}(i) - a_k)^- \leq \frac{\lambda - \chi}{2}.$$

Lemma 1.3 ([11],Lemma 2). *Let $||\mathcal{A}|| < \infty$ and $\lim_n \sup_i a_{nk}(i) = 0$. Then, there exists a $y \in \ell_\infty$ with $||y|| \leq 1$ such that*

$$\limsup_n \sup_i \sum_k a_{nk}(i) y_k = \limsup_n \sup_i \sum_k |a_{nk}(i)|. \tag{1.7}$$

2. The Main results

Theorem 2.1. *Let A be $\sigma_{B(r,s)}$-conservative. Then, for some constant $\lambda \geq |\chi_{B(r,s)}|$ and for all $x \in \ell_\infty$,*

$$\limsup_m \sup_n \sum_k \left(\widetilde{a}(m,n,k) - \alpha_k\right)x_k \leq \frac{\lambda + \chi_{B(r,s)}}{2} L(x) - \frac{\lambda - \chi_{B(r,s)}}{2} \ell(x) \tag{2.1}$$

if and only if

$$\limsup_m \sum_k \left|\widetilde{a}(m,n,k) - \alpha_k\right| \leq \lambda \text{ uniformly in } n. \tag{2.2}$$

Where $\widetilde{a}(m,n,k) = \frac{1}{m+1}\sum_{j=0}^{m} sa_{\sigma^j(n)-1,k} + ra_{\sigma^j(n),k}$.

Proof. Necessity: Suppose that (2.1) holds and, if we define the matrix by $\mathcal{C} = (c_{mk}(n))$ by

$$c_{mk}(n) = \Big(\frac{1}{m+1}\sum_{j=0}^{m} sa_{\sigma^j(n)-1,k} + ra_{\sigma^j(n),k} - \alpha_k\Big). \tag{2.3}$$

for all m,n,k. Then, since A is $\sigma_{B(r,s)}$- conservative, the matrix $\mathcal{C}$ satisfies the hypothesis conditions of the Lemma 1.3. Hence for a $y \in \ell_\infty$ such that $\| y \| \leq 1$ we have (1.7) for matrix $\mathcal{C}$. Hence, using (2.1), we can write

$$\begin{aligned}
\limsup_m \sup_n \sum_k |c_{mk}(n)| &= \limsup_m \sup_n \sum_k c_{mk}(n) y_k \\
&\leq \frac{\lambda + \chi_{B(r,s)}}{2} L(y) - \frac{\lambda - \chi_{B(r,s)}}{2} \ell(y) \\
&\leq \Big(\frac{\lambda + \chi_{B(r,s)}}{2} + \frac{\lambda - \chi_{B(r,s)}}{2}\Big) ||y|| \\
&= \lambda.
\end{aligned}$$

Thus, we have the condition (2.2).

Sufficiency: Let (2.2) holds and $x \in \ell_\infty$. Then, for any given $\varepsilon > 0$, we can write $\ell(x) - \varepsilon \leq x_k \leq L(x) + \varepsilon$ whenever $k \geq k_0$. Now, can write

$$\sum_k c_{mk}(n) x_k = \sum_{k<k_0} c_{mk}(n) x_k + \sum_{k \geq k_0} c_{mk}(n)^+ x_k - \sum_{k \geq k_0} c_{mk}(n)^- x_k.$$

Hence, since A is $\sigma_{B(r,s)}$-conservative, we get by that Lemma 1.2 that

$$\begin{aligned}
\limsup_m \sup_n \sum_k c_{mk}(n) x_k &\leq \frac{\lambda + \chi_{B(r,s)}}{2}\big(L(x) + \varepsilon\big) - \frac{\lambda - \chi_{B(r,s)}}{2}\big(\ell(x) - \varepsilon\big) \\
&= \frac{\lambda + \chi_{B(r,s)}}{2} L(x) - \frac{\lambda - \chi_{B(r,s)}}{2} \ell(x) + \lambda\varepsilon.
\end{aligned}$$

This completes the proof since ϵ is arbitrary and for all $x \in \ell_\infty$. □

Theorem 2.2. *Let A be $\sigma_{B(r,s)}$-conservative. Then, for some constant $\lambda \geq \left|\chi_{B(r,s)}\right|$ and for all $x \in \ell_\infty$,*

$$\limsup_m \sup_n \sum_k \big(\widetilde{a}(m,n,k) - \alpha_k\big) x_k \leq \frac{\lambda + \chi_{B(r,s)}}{2}\beta(x) + \frac{\lambda - \chi_{B(r,s)}}{2}\alpha(-x) \tag{2.4}$$

if and only if (2.2) holds and

$$\lim_m \sum_{k \in E} \left|\widetilde{a}(m,n,k) - \alpha_k\right| = 0 \text{ uniformly in } n \tag{2.5}$$

for every $E \subset \mathbb{N}$ with $\delta(E) = 0$, where $\beta(x) = st - \limsup x_k$.

Proof. Necessity: Let (2.4) hold. Then, by attention hypothesis, we can write $\beta(x) \leq L(x)$ and $\alpha(-x) \leq -\ell(x)$. Thus, the necessity of the condition (2.2) follows from Theorem 2.1. Now let us show the necessity of (2.5). For any $E \subseteq \mathbb{N}$ with $\delta(E) = 0$, let us define a matrix $\mathcal{T} = (t_{mk}(n))$ as follows

$$t_{mk}(n) = \begin{cases} c_{mk}(n) & , \quad k \in E \\ 0 & , \quad k \notin E. \end{cases}$$

Then, since A is $\sigma_{B(r,s)}$-conservative, it is clear that $\mathcal{T}$ satisfies the conditions of Lemma 1.2 and hence there exists a $y \in \ell_\infty$ such that $\| y \| \leq 1$ and

$$\limsup_m \sup_n \sum_k t_{mk}(n) y_k = \limsup_m \sup_n \sum_k |a_{mk}(n)|.$$

Thus, for the same E, let use choose the sequence (z_k) by

$$z_k = \begin{cases} 1 & , \quad k \in E \\ 0 & , \quad k \notin E. \end{cases}$$

Then, clearly $z \in st_0$ and so, $\beta(z) = \alpha(z) = st - \lim z = 0$. Hence, by the assumption and (1.7), we get that

$$\begin{aligned} \limsup_m \sup_n \sum_{k \in E} |t_{mk}(n)| &\leq \frac{\lambda + \chi_{B(r,s)}}{2} \beta(z) + \frac{\lambda - \chi_{B(r,s)}}{2} \alpha(-z) \\ &= 0 \end{aligned}$$

which implies the necessity of the condition (2.5).

Sufficiency: Assume that (2.2) and (2.5) hold. For any $x \in \ell_\infty$, let us define $E_1 = \{k : x_k > \beta(x) + \varepsilon\}$ and $E_2 = \{k : x_k < \alpha(x) - \varepsilon\}$. Then $\delta(E_1) = \delta(E_2) = 0$, [28]. Hence the set $E = E_1 \cap E_2$ has also zero density and

$$\alpha(x) - \varepsilon \leq x_k \leq \beta(x) + \varepsilon \tag{2.6}$$

whenever $k \notin E$. Now; it can be written that

$$\sum_k c_{mk}(p) x_k = \sum_{k \in E} c_{mk}(p) x_k + \sum_{k \notin E} c_{mk}(p)^+ x_k - \sum_{k \notin E} c_{mk}(p)^- x_k.$$

Thus, since (2.5) implies that the first sum on the right hand-side is zero, by Lemma 1.2 and from (2.6), we get

$$\begin{aligned} \limsup_m \sup_n \sum_k c_{mk}(n) x_k &\leq \frac{\lambda + \chi_{B(r,s)}}{2} \big(\beta(x) + \varepsilon\big) + \frac{\lambda - \chi_{B(r,s)}}{2} \big(\alpha(-x) - \varepsilon\big) \\ &= \frac{\lambda + \chi_{B(r,s)}}{2} \beta(x) + \frac{\lambda - \chi_{B(r,s)}}{2} \alpha(-x) + \lambda\varepsilon. \end{aligned}$$

Since ε is arbitrary, this completes the proof. □

Theorem 2.3. *Let A be $\sigma_{B(r,s)}$-conservative. Then, for some constant $\lambda \geq |\chi_{B(r,s)}|$ and for all $x \in \ell_\infty$,*

$$\limsup_m \sup_n \sum_k \big(\widetilde{a}(m,n,k) - \alpha_k\big) x_k \leq \frac{\lambda + \chi_{B(r,s)}}{2} q_\sigma(x) + \frac{\lambda - \chi_{B(r,s)}}{2} q_\sigma(-x) \tag{2.7}$$

if and only if (2.2) holds and

$$\lim_m \sum_k \big| c_{mk}(n) - c_{m,\sigma(k)}(n) - (\alpha_k - \alpha_{\sigma(k)}) \big| = 0 \text{ uniformly in } n. \tag{2.8}$$

Proof. Necessity: Suppose that (2.7) holds. Then, since $A \in (c, \widehat{V_\sigma})$, we can be written $q_\sigma(x) \leq L(x)$ and $q_\sigma(-x) \leq -\ell(x)$ for all $x \in \ell_\infty$, thus the necessity of (2.2) follows from Theorem 2.1. If define $\mathcal{R} = (r_{mk}(n))$ by $r_{mk}(n) = \big(c_{mk}(n) - c_{m,\sigma(k)}(n)\big)$, we have (1.3) for $\mathcal{R}$.

Let us choose z such that $z_k = 0$, $k \notin \sigma(\mathbb{N})$. Hence, since $(z_k - z_{\sigma(k)}) \in Z$, (2.7) implies that

$$\begin{aligned}
\limsup_m \sum_k |r_{mk}(n)| &= \limsup_m \sum_k r_{mk}(n) z_{\sigma(k)} \\
&= \limsup_m \sum_k c_{mk}(n)(z_k - z_{\sigma(k)}) \\
&\leq \frac{\lambda + \chi_{B(r,s)}}{2} q_\sigma(z_k - z_{\sigma(k)}) + \frac{\lambda - \chi_{B(r,s)}}{2} q_\sigma(z_{\sigma(k)} - z_k) \\
&= 0
\end{aligned}$$

which is (2.8).

Sufficiency: Let the condition (2.2) and (2.8) hold. By the same argument as in Theorem 23 of [33], one can easily see that for any $x \in \ell_\infty$

$$\sum_k c_{mk}(n)(x_k - x_{\sigma(k)}) = \sum_k r_{mk}(n) x_{\sigma(k)}$$

where the matrices $\mathcal{C}$ and $\mathcal{R}$ are as above.

Hence, since $(x_k - x_{\sigma(k)}) \in Z$, (2.8) implies that $\mathcal{C} \in (Z, S_0 \cap \ell_\infty)$. We also see from the assumption that (2.1) holds. Thus, taking infimum over $z \in Z$ in (2.1) we get that

$$\begin{aligned}
\inf_{z \in Z} \left(\limsup_m \sup_n \sum_k c_{mk}(n)(x_k + z_k) \right) &\leq \frac{\lambda + \chi_{B(r,s)}}{2} L(x+z) - \frac{\lambda - \chi_{B(r,s)}}{2} \ell(x+z) \\
&= \frac{\lambda + \chi_{B(r,s)}}{2} W(x) + \frac{\lambda - \chi_{B(r,s)}}{2} W(-x).
\end{aligned}$$

On the other hand, since $\sigma_{B(r,s)} - \lim \mathcal{C} z = 0$ for $z \in Z$,

$$\begin{aligned}
\inf_{z \in Z} \left(\limsup_m \sup_n \sum_k c_{mk}(n)(x_k + z_k) \right) &\geq \limsup_m \sup_n \sum_k c_{mk}(n) x_k + \inf_{z \in Z} \left(\limsup_m \sup_n \sum_k c_{mk}(n) z_k \right) \\
&= \limsup_m \sup_n \sum_k c_{mk}(n) x_k.
\end{aligned}$$

Since $W(x) = q_\sigma(x)$ [32] for all $x \in \ell_\infty$, we conclude that (2.7) holds and the proof is completed.

□

3. CONCLUSION

Lorentz [12] introduced the concept of almost convergence in 1948, and Das [11] has characterized a class of conservative matrices in terms of inequalities involving sublinear functionals on ℓ_∞.

Schaefer [34] and Fast [13] defined σ-convergent sequences and statistically convergent ,respectively. Çoşkun and Çakan [15] determined a class of conservative, σ-conservative and statistically(st)-conservative matrices using the same technique.

Furthermore, quite recently, Kirişçi and Başar [21] introduced the new sequence spaces $\hat{f}$ derived from the spaces f of almost convergent sequences by means of the domain of the generalized difference bant matrix $B(r,s)$.

In this paper, we have determined $\widehat{V_\sigma}$ spaces and $\sigma_{B(r,s)}$ -conservative matrices studying some inequalities which are analogous to the famous Knopp's theorem. Our corresponding results are much more general than given by Çoşkun and Çakan in, [15]. So, the main results of the present paper fill up the gap in the existing literature.

Competing interests
The author declares that they have no competing interests.

Authors Contributions
In the preparation of this article, the contributions of the authors are equally. All authors read and approved the final manuscript.

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