Journal of Inequalities and Special Functions
ISSN: 2217-4303, URL: http://www.ilirias.com
Volume 8 Issue 1(2017), Pages 1-12.

ON PROFESSOR IVAN DIMOVSKI CONTRIBUTIONS

VIRGINIA KIRYAKOVA

Abstract. This special issue of the journal JIASF is dedicated to Professor Ivan Dimovski, Member of Editorial Board, and his contributions in mathematical analysis and its applications. The basic, widely World known and explored of them, can be classified in the following topics: operational calculus, integral transforms, theory of multipliers of convolutional algebras, expansions in eigenfunctions and associated functions, explicit characterization of commutants and automorphisms in them, linear nonlocal boundary value problems for differential equations of mathematical physics. Shortly, he is known as the father of the Convolutional Calculus, as extension of the Mikusinski operational calculus. Besides, he has important role in Bulgarian mathematics, by creating a school of disciples, contribution to education in mathematics both in secondary schools and universities, translating many books and works from foreign languages into Bulgarian and vice versa.

1. Short biographical data

Professor Dr.Sc. Ivan Dimovski, Institute of Mathematics and Informatics - Bulgarian Academy of Sciences (B.A.S.), Corresponding Member of B.A.S., and a *Member of Editorial Board of "Journal of Inequalities and Special Functions"*, is born on July 7, 1934 in the Bulgarian village Oreshak.

Prof. Dimovski started his successful mathematical carrier yet as a pupil in the secondary school in the town of Troyan, near to his born village, when he received a prize as a winner in the first national mathematical olympiad in Bulgaria. Then he graduated at the Mathematics Dept. at Sofia University, and another reason to be proud, was his active participation in the famous seminar of Prof. Jaroslav Tagamlitski on topics of real analysis, where many other students took part, later becoming known scholars. There has been also a period in his studies and research interests dedicated to Mechanics.

His carrier as a teacher and assistant professor started in the town of Rousse, at a secondary school and then in the high school there. Then it continued in Sofia in the Institute of Mathematics of Bulgarian Academy Sciences, till nowadays as an Emeritus Prof. and Honorary member of Institute - starting from a young mathematician (1959); through full professor (1982); a long-year chief of "Complex Analysis" department (1986-2004). He had been also a lecturer in many Bulgarian universities

2000 *Mathematics Subject Classification.* 44A40, 44A35, 44A15, 34B10, 47B47.
Key words and phrases. Convolutional calculus; Mikusinski operational calculus; Obrechkoff integral transform; nonlocal boundary value problems; commutants of linear operators.

Submitted December 21, 2016. Published January 7, 2017.
This paper is performed in the frames of the Bilateral Res. Projects 'Analytical and numerical methods for differential and integral equations and mathematical models of arbitrary ... order' between BAS and SANU, and 'Analysis, Geometry and Topology' between BAS and MANU.

on large variety of courses (calculus, history of mathematics, operational calculus, potential theory, theory of elasticity and continuum mechanics); and President of Scientific Council on "Applied Mathematics and Mechanics"(2005-2009). In 1997, Prof. Dimovski was elected as a corresponding member of Bulgarian Academy of Sciences.

Prof. Dimovski is author of more than 120 scientific papers, and of the *monograph* [2] "*Convolutional Calculi*" (1990), Kluwer (now available as e-book by Springer), referred to by other authors more than 800 times. Many papers, theses and monographs of his Bulgarian collaborators and of foreign authors are inspired by his ideas and results and are using them essentially, some of them even containing his name in the titles or in the names of new mathematical notions.

He has been an invited speaker at many international conferences and visiting professor in foreign universities, as in: Russia, Germany, Serbia, Poland, Spain, Venezuela, Kuwait, Macedonia, etc.; member of the Editorial Boards of the international journals "*Integral Transforms and Special Functions*", "*Fractional Calculus and Applied Analysis*" and "*Special Functions and Inequalities*"; reviewer for many other international journals and publishers; organizer of a series of international mathematical conferences.

Among his contributions is the creation of a *Bulgarian mathematical school* (consisting of 11 Ph.D. students, many M.Sc. students and research collaborators), as well as a group of foreign collaborators and followers, in the field of Operational Calculus (extended by him as Convolutional Calculus) and Integral Transforms and their applications in Applied Analysis, Local and Nonlocal Boundary Value Problems, Fractional Calculus.

The list of *his PhD students* contains: Prof. Dr.Sc. Nikolai Bozhinov; Prof. Dr.Sc. Sava Grozdev; Asso.Prof. Dr. Radka Petrova; Asso.Prof. Dr. Stefan Koprinski; Dr. Dimiter Mineff; Prof. Dr.Sc. Virginia Kiryakova; Assist. Prof. Mladen Vassilev; Asso.Prof. Dr. Emilia Bazhlekova; Asso.Prof. Dr. Svetlana Mincheva-Kaminska; Asso.Prof. Dr. Margarita Spiridonova; Asso.Prof. Dr. Yulian Tsankov.

It should be noted that Dimovski's theory and ideas have been used also in master and doctoral theses and great number of papers by foreign authors, sometimes his name appearing even in the titles of such works.

His name and achievements, mainly as a "father of the contemporary convolutional calculus" are widely known to the World mathematical community.

In his young years, Prof. Dimovski *translated more than 50 famous mathematical books* from foreign languages (English, German, Russian and French) into Bulgarian, and thus he made many brilliant mathematical masterpieces available to the audience of Bulgarian mathematicians. Vice versa, he translated from Bulgarian to English or German some works of famous Bulgarian mathematicians. Another trend of his activities is the field of *"School Mathematics"* - as a lecturer for talented pupils prepared for mathematical olympiads, and as author of many classroom books on elementary mathematics, agendas and school curricula.

For his scientific achievements and contributions to Bulgarian science, Prof. Ivan Dimovski has been awarded the academic price on the name of Bulgarian famous mathematician "Nikola Obrechkoff" (1979), the honor medal "Marin Drinov" of Bulgarian Academy of Sciences (2004), and is elected Honorary member of Institute.

2. Short annotation of Prof. Dimovski's scientific contributions

The key role in Prof. Dimovski's studies, from his very first publication [6] to the recent ones, is played by the term *"convolution of a linear operator"*. A new notion in mathematics usually deserves a long-standing role, only if it helps to solve problems that can be formulated without its use, but their solution requires an essential use of it. Such a notion happens to be the one introduced by Dimovski, as the base of his "*convolutional approach*". By means of this approach, he has built new operational calculi for local and nonlocal boundary value problems, extending the area of applicability of the multipliers theory and relating it to the theory of commuting linear operators. Based on this convolutional approach, a *new variant of the Duhamel principle* has been developed, for a large variety of important nonlocal BVPs for equations of mathematical physics.

The classical operational calculus of Jan Mikusinski is based on the well known convolution of Duhamel

$$f * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau. \tag{1}$$

Yet in his first scientific paper [6] of 1962, Dimovski proved that putting on the base of the operational calculus any other continuous convolution of the integration operator, this leads to another operational calculus, isomorphic to Mikusinski's one. The idea to generalize the direct algebraic approach of Mikusinski for building operational calculi for other operators different from the integration operator, has encountered both *conceptual and technical problems*, yet in the first attempts made by some Russian, German and Hungarian mathematicians. Dimovski's notion "*convolution of a linear operator*" opened the way to such generalizations, allowing to speak about "operational calcul*I*" (in *plural* form). Its origin is hidden yet in paper [6] and formulated for some particular operators and spaces in [28], with the most important examples for convolutions proposed in [11], [12], [21], to justify its general character. Since in each particular case of an operator, one needs to solve the problem of constructing a convolution in explicit form, the happy hint for Dimovski has been to start (1966-1974) with a very general class of differential operators of Bessel type of arbitrary order $m \geq 2$ (nowadays called *"hyper-Bessel operators"*), and then to continue with the general linear differential operators of first and second order. The whole theory with the known applications (by then) can be found in his monograph [2]: Ivan Dimovski, *Convolutional Calculus*, Kluwer Academic Publishers, Dodrecht - Boston - London, 1990 (its first edition being by Printing House of Bulg. Acad. Sci. in 1982; nowadays available as e-book by Springer).

Dimovski's basic definition in his convolutional calculus is the following: *A bilinear, commutative and associative operation* $* : X \times X \mapsto X$ *is called a convolution of the linear operator* $L : X \mapsto X$*, mapping a given linear space* X *into itself, when* $L(f * g) = (Lf) * g$ *for all* $f, g \in X$.

To find a convolution of a given linear operator in explicit form, as a rule, is a difficult and nontrivial task. However, once found, a convolution operation allows easily to build an operational calculus for the corresponding operator, and thus to solve the basic spectral problems, related to it: finding various spectral functions of this operator, and also of its commutant. In this respect, the paper [4] of 1978 is important, including a fact unknown by then: if such an operator has a cyclic element, then the rings of the multipliers of a nontrivial convolution of it, and of the operators commuting with it, *coincide*! This allowed Dimovski to find an elegant explicit characterization of the linear operators $M : C[0,1] \mapsto C[0,1]$, commuting with the classical integration operator $Lf(t) = \int\limits_0^t f(\tau)d\tau$. In his monograph [2] it is proven that for the commutation of M and l it is necessary and sufficient that M has an integral representation of the form $Mf(t) = \dfrac{d}{dt}\int\limits_0^t f(t-\tau)\alpha(\tau)d\tau$, where $\alpha(\tau)$ is a function simultaneously continuous and with bounded variation. This seems to be a result of wide mathematical importance.

The main approach in finding new convolutions, using some already known basic ones, happened to be the "*similarity method*", called also a "*method of transmutations*". For the first time, Dimovski formulated this idea in [3] in 1974, and with good justifying examples, in [5]. The method of similarity is based on the following simple fact: If $T : X \mapsto \widehat{X}$ and $\widehat{*}$ is a convolution of the linear operator $\widehat{L} : \widehat{X} \mapsto \widehat{X}$, then the operation $f * g := T^{-1}[(Tf)\widehat{*}(Tg)]$ is a convolution for the linear operator $L = T^{-1}\widehat{L}T$, mapping X into X. Usually, T is called a transmutation or similarity operator, and L and $\widehat{L}$ are similar operators.

Dimovski used essentially this approach, for the first time, in the case of the *Bessel-type operators*, [42], [31], [32], [43]. The corresponding similarity operator T found by him, is a generalization of the classical transformations of Poisson and Sonine (nowadays, we use to call it a *"Poisson-Sonine-Dimovski" transformation*). Next, by means of the similarity operators of Delsarte-Povzner, he proved the possibility for building operational calculi not only for initial value problems for the general 2nd order differential operator ([22]) but also for a wide class of nonlocal boundary value problems, including as a very special case the famous Sturm-Liouville problem ([16], [17], [26], [27]).

From a point of view of wide-range mathematics, Dimovski himself considers as most valuable the *contributions related to the spectral theory of the classical (1st order) differential operator*. In the 30's of the 20th century, the French mathematician Jean Delsarte (the founder of "Bourbaki" group) made several unsuccessful attempts to find a convolution, related to the most general spectral problem for the differentiation operator. His only achievement then was the generalization of the classical Taylor formula for this operator. The general spectral problem for the differentiation operator (in a corresponding space) consists in studying the resolvent operator L_λ, the result of its action, $y = L_\lambda f$ being considered as a solution of the nonlocal boundary value problem (BVP) $y' - \lambda y = f$, $\Phi(y) = 0$, where Φ is an arbitrary nonzero linear functional.

Dimovski solved the problem of Delsarte in 1974 (papers [7], [8]) when he found and proved that the operation

$$(f * g)(t) = \Phi_\tau \left\{ \int\limits_\tau^t f(t + \tau - \sigma)g(\sigma)d\sigma \right\} \tag{2}$$

is a convolution of the resolvent L_λ, and built the respective operational calculus ([9]). The same convolution was rediscovered independently, by the German mathematician Lothar Berg (member of German Academy of Sciences) in 1976, who made reference to Dimovski's paper of 1974, thus acknowledging his priority. This convolution (called now as *Dimovski-Berg convolution*) allowed to obtain a complete solution of *the problem for multipliers of the Leontiev extensions in exponentials in the complex domain* (papers [10], [18]).

Interesting analogues of the differentiation operator are *the operator for backward shift translation* $\Delta f(t) = \left(f(t) - g(t)\right)/t$ (called also the Pommiez operator) and *the finite-difference operator in the space of sequences.* It was confirmed that Dimovski's general scheme works successfully also in these two cases (see [17], [14], [15]). The experts acknowledged strongly also the results on *finding commutant of the Gelfond-Leontiev integration operator*, from joint papers (with Kiryakova) as [19], [20], referred to in the books by M.K. Fage and N.I. Nagnibida, "Problem of Equivalency of Ordinary Differential Operators", Nauka, Novosibirsk, 1987 (In Russian) and by S.G. Samko, A.A. Kilbas and O.I. Marichev, "Fractional Integrals and Derivatives", 1987 (Nauka) and 1993 (Gordon and Breach).

From the point of view of applied mathematics, the most important Dimovski's results concern the *convolutions for nonlocal BVP for 2nd order linear differential operators*, since these operators play basic role in the *problems of mathematical physics.* In the mathematical literature, there has been a lack of a well-developed theory of nonlocal BV problems. Each of the few authors studying such BVPs, has considered some very special cases, without any general view on these kind of problems. As to the local BVPs of mathematical physics, the most popular method for their solving is the Fourier method. Unfortunately, this method hardly allows a computer realization. Concerning the widely known difference methods, the experts confess that "taking into account the subsequent principle of calculation of the contemporary computers, such an approach requires a .... expense of time" (e.g. V.Z. Alad'ev, M.L. Shishakov, "Automated Working Space of a Mathematician", Moscow, 2000 (in Russian), p. 644). Anyway, till recently it seems nobody has tried to solve, by means of commonly used PCs, serious local and nonlocal BV problems related to equations of mathematical physics. Dimovski has proved, at least in principle, the possibility of using the "*convolutional method*" for such a task. The essence of his approach consists in *combining the Fourier method with the Duhamel principle.* The weakness of the Fourier method is in the necessity to use expansions of the boundary functions in series of eigenfunctions. This procedure requires calculating of number of integrals (from dozens to hundreds) and afterwards, summing the series obtained as solution in many points. In his works [21], [27], [23], [2], Dimovski proposes convolutions for BV problems for Sturm-Liouville operators with one local, and another - in general - nonlocal boundary value conditions, and thus opened the way to extend the Duhamel principle from a time-variable to space-variables in linear problems of mathematical physics. Generally speaking, the Duhamel principle consists in finding all solutions of a BVP by means of one particular solution of same problem. Such a particular solution is defined in terms of simple boundary functions and does not require numerical computation of definite integrals. Combining the Fourier method with the Duhamel principle allows to avoid two hard stages, from computing point of view, in the realization of Fourier method: expansion of the boundary functions in series of eigen- and associated functions, and the summation in many points of the obtained solution which is a rule, a slow convergent series. The numerical experiments ([49]) show high efficiency of such a convolutional approach. A natural question arises about "If the combination of these two methods is so efficient both in theoretical and computational aspects, why nobody had the hint before, to use it?" Possibly, the answer is that nobody had expected the existence of a simple explicit expression for the corresponding convolutions of [21] ... Some examples of realization of this Dimovski's schema, can be found in the papers by his disciples, E. Bazhlekova-I.Bazhlekov and Yu. Tsankov, *published in this special JIASF issue.*

Another application of Dimovski's convolutions for nonlocal BVPs for 2nd order linear ordinary differential operators, is the *possibility to generalize the notion of finite integral transformation for each of the corresponding BV problems* ([26], [27]). This solves automatically also the problem of obtaining of explicit convolutions for these finite integral transforms. As a special case, it is obtained a solution of the problem posed in 1972 by Churchill, to find convolutions of the finite Sturm-Liouville transformations (see Dimovski's monograph [2]). As a by-side product of the convolutions related to the general Bessel-type operators (paper [29]) it is found an explicit convolution of the classical Meijer transform (paper [24]). Another well appreciated Dimovski's result is the explicit convolution of the discrete Hermite convolution ([25]). A whole chapter is dedicated to this result in the book "*Integral Transforms and Their Applications*" by L. Debnath, one of the pioneers in searching for such a convolution.

From the point of view of Bulgarian mathematics and its traditions, the most important Dimovski's contribution is the identification, studying and giving an international popularity to the so-called "*Obrechoff integral transform*", nowadays being a widely popular generalization of the Laplace transform ([8], [9], [24], etc). For his achievements on the subject, Dimovski was awarded in 1979 with the "Nikola Obrechkoff" Prize of Bulgarian Academy of Sciences. The Bulgarian mathematician Nikola Obrechkoff (1896-1963) himself never claimed for an authorship of a new integral transform but only for a formula for integral representation of functions on the real half-axis, *as an extension of a result of S. Bernstein.* In 60's-70's Dimovski studied the so-called general Bessel type differential operators, i.e. the singular differential operators of arbitrary order $m \geq 2$ naturally extending the 2nd order Bessel operator,

$$B = t^{\alpha_0}\frac{d}{dt}t^{\alpha_1}\frac{d}{dt}t^{\alpha_2}\dots\frac{d}{dt}t^{\alpha_m} = t^{-\beta}\left(t\frac{d}{dt}+\beta\gamma_1\right)\dots\left(t\frac{d}{dt}+\beta\gamma_m\right)$$
$$= t^{-\beta}\left[t^m\frac{d^m}{dt^m}+a_1t^{m-1}\frac{d^{m-1}}{dt^{m-1}}+\dots+a_{m-1}t\frac{d}{dt}+a_m\right], \quad 0<t<\infty, \qquad (3)$$

nowadays known as "*hyper-Bessel differential operators*". Developing operational calculi for the corresponding integral operators L, initial right inverse to B ($BL = I$), he had the idea that the Obrechkoff integral transform of 1958 (a slight modification of it) can be used successfully as a transform basis, in the same way as the Laplace transform is used in the classical operational calculus for the usual differentiation/ integration operators. Later on, the studies on the Obrechkoff transform and on the hyper-Bessel operators have been prolonged in some joint papers of Dimovski and Kiryakova, e.g. [33]–[37], [39] and extended to a theory of *generalized fractional calculus* and related to important *classes of special functions* in the monograph: V. Kiryakova, *"Generalized Fractional Calculus and Applications"*, Longman - J. Wiley, 1994. The keys of these further developments were given by: considering the *fractional powers of the hyper-Bessel operators* as the simplest "generalized operators of fractional integration and differentiation"; and by the role of the Meijer G-functions as kernel-functions of the Obrechkoff transform and of the hyper-Bessel integral operator, as well as solutions of classes of hyper-Bessel differential equations. For the details on this matter, see the survey paper: V. Kiryakova, *From the hyper-Bessel operators of Dimovski to the generalized fractional calculus*, Fract. Calc. Appl. Anal. **17**, 4 (2014) 977–1000; doi: 10.2478/s13540-014-0210-4.

Another important result, related to the Bessel type operators of arbitrary order, is the very far *generalization of the classical transmutation operators of Sonine and Poisson* (Dimovski's papers [40]–[42], [31], [38]), by means of which the equivalency of every two Bessel type operators of one and same order is proven. For the applications, the generalized Sonine operator is important, since it transforms an arbitrary Bessel-type differential operator of order m into the m-tuple differentiation $(d/dt)^m$. It is worth mentioning that in each particular case, the corresponding Dimovski's formula gives the best possible result. *This is a solution of the Delsarte problem*, posed by him in a manuscript (about 80 p.) published only posthumously in 1970 in the 2nd volume of his collected papers. And the Poisson-Dimovski transmutation and its generalizations in terms of fractional calculus, allowed Kiryakova to give explicit solutions to hyper-Bessel equations of arbitrary integer order and to rather general class of differential equations of fractional multi-order, bu reducing them to simpler equations with known solutions. Some more detailed comments and examples on Dimovski's transmutation operators can be seen in the papers by S. Sitnik, *published in this special JIASF issue.*

There exists a field of mathematics, which would look like quite different today if there were not the contributions of Dimovski, the *"Operational Calculus"*, nowadays extended to so-called *"Convolutional Calculus"*. In "Mathematics Subject Classification" it is classified as a section A44. It leads its origin from the studies of O. Heaviside in the end of 19th century, and remained without a strong mathematical formulation by the 50's of 20th century, when the Polish mathematician Jan Mikusinski proposed his direct algebraical approach based on the classical Duhamel convolution (1), thus justifying the Heaviside calculus. Predecessors of Mikusinski were Volterra and Pérèz (1924, 1943) to whom belonged the idea of using convolutional fractions for the same purpose. In 1957 the Russian mathematician V.A. Ditkin gave an example of operational calculus, different from Mikusinski's one. Namely, while Mikusinski's calculus is concerned with the Cauchy problem for the differentiation operator d/dt, Ditkin's calculus concerns the same problem but for the simplest operator of Bessel type, $(d/dt)t(d/dt)$. New examples of operational calculi of Bessel type of rather particular forms appeared in the 60's, by different authors. In the papers [13], [28]–[31], Dimovski established the applicability of Mikusinski's approach to the most general Bessel-type operator (3). In [30] and [42] he proved, for the first time, that the operational calculi for all Bessel-type operators in the Mukusinski scheme, are isomorphic. Especially, they are isomorphic to the Mikusinski operational calculus, since the classical differential operator is also a Bessel-type operator. In a paper [22] joint with N. Bozhinov, Dimovski proved that the operational calculi for initial value problems for the general 2nd order linear differential operator are also isomorphic to Mikusinski's calculus.

Some of the studies by Dimovski's scheme have been extended also to classes of differential-difference operators, like the Dunkl operator. For commutants and nonlocal operational calculi for this operator, see for example his papers [47], [50]. More on the theory of the Dunkl operator can be seen in the paper by Kh. Trimeche, *published in this special JIASF issue.*

Conceptually new are *Dimovski's contributions for building operational calculi for boundary value problems* (BVPs) *and* especially, *for nonlocal BVPs for linear differential operators of 1st and 2nd order.* In contrary to the "Algebraic Analysis" of D. Przeworska-Rolewicz and to the approach of P. Bittner, in which two algebraic systems are considered - a ring of operators and a linear space, *in Dimovski's scheme it is considered a single algebraic system* - the ring of the multiplier quotients. This approach was described first in the Dr.Sc. thesis [1] of Dimovski and then, in his monograph [2]. *The advantage of using multipliers quotients instead of convolutional ones*, as it is in Mukisinski's approach, can be seen in the operational calculi of functions of several variables (papers e.g. [44], [45], [46]). The more, there exist cases (*resonance cases*) when the convolutional quotients' approach is not applicable (see [12], [13], [61]), while the multipliers' one works successfully.

To summarize, Dimovski's basic contributions can be classified in the following domains of mathematical analysis: operational calculus, integral transforms, theory of multipliers of convolutional algebras, expansions in eigenfunctions and associated functions, explicit characterization of commutants and automorphisms in them, linear nonlocal boundary value problems for equations of mathematical physics.

The recent papers of Dimovski and his collaborators show the continuation and further developments and applications of his approach and ideas, some few of them (updated till 2010) can be seen at http://degruyteropen.com/people/dimovski/.

3. Some publications of Ivan Dimovski

[1] *Convolutional Method in Operational Calculus*, Summary of D.Sc. Thesis, Sofia (1977).

[2] *Convolutional Calculus.* Kluwer Academic Publishers, Dordrecht-Boston-London (1990); First publ.: Publ. House of Bulg. Acad. Sci., Sofia (1982); Springer e-book at http://link.springer.com/book/10.1007/978-94-009-0527-6; doi: 10.1007/978-94-009-0527-6.

Scientific Articles

1. Some Popular Papers and Surveys

[3] *On the bases of operational calculus*, in: "Mathematics and Math. Education. Talks of 2nd Spring Conf. UBM" (1974) 103–112 (In Bulgarian).

[4] *Convolutions of right inverse operators and representation of their multipliers*, Compt. rend. Acad. bulg. Sci. **31**, 11 (1978) 1377–1380.

[5] *The convolutional method in operational calculus*, in: Generalized Functions and Operational Calculus. Proc. Varna Conf. '1975 (1979) 69–68.

2. Convolutions, Operational Calculi, Multipliers and Commutants Related to Differentiation Operators

[6] *Uniqueness of the Mikusinski field*, Fiz.-matemat. spisanie **5**, 1 (1962) 56–59 (In Bulgarian).

[7] *On an operational calculus for vector-valued functions.* Math. Balkanica **4** (1974) 129–135.

[8] *Convolutions for the right-inverse operators of the general linear differential operator of the first order*, Serdica **2** (1976) 82–86.

[9] *Nonlocal operational calculi*, in: Trudy Matemat. Instituta im. V.A.. Steklova, **203** (1993) 58–73 (In Russian); Engl. Transl.: *Nonlocal operational calculi*, Proc. Steklov Inst. Math. (1995), Issue 3, 53–65.

[10] *The finite Leontiev transform*, Pliska **4** (1981) 102–109.
[11] *Holomorphic convolutions and their uses*, in: Complex Analysis and Appls. Varna '81, Sofia (1984) 138–147.
[12] *Mean-periodic operational calculi* (Dimovski, I., Skrnik, K.), in: Algebraic Analysis and Related Topics. Banach Center Publications **53** (2000) 105–112.
[13] *Mean-periodic operational calculi. Singular cases* (Dimovski I., Skornik, K.), Fract. Calc. Appl. Anal. **4**, 2 (2001) 237–243.
[14] *Discrete operational calculi for two-sided sequences* (Dimovski I., Kiryakova V.), The Fibonacci Quarterly **5** (1992) 159–168.
[15] *Generalizations of finite and discrete Fourier transforms*, in: "Proc. International Workshop on Recent Advances in Applied Mathematics (RAAM), Kuwait" (1996) 101–105.
[16] *Representation of operators which commute with differentiation in an invariant hyperplane*, *Compt. rend. Acad. bulg. Sci.* **31**, 10 (1978) 1245–1248.
[17] *Convolutions, multipliers and commutants for the backward shift operator* (Dimovski I., Mineff D.), Pliska **4** (1981) 128–136.
[18] *Convolutions, multipliers and commutants connected with multiple Dirichlet expansions* (Dimovski I., Bozhinov N.), *Serdica. Bulg. Math. Publ.* **9** (1983).
[19] *Convolution representation of the commutant of Gelfond-Leontiev integration operator*, Compt. rend. Acad. bulg. Sci. **34**, 12 (1981) 1643–1646.
[20] *Representation formulas for the commutants of integer powers or Gelfond-Leontiev integration operators*, in: "Mathematics and Math. Education. Talks of 11th Spring Conf. UBM" (1982) 166–172.

3. Convolutions and Finite Integral Transforms for Second Order Linear Differential Operators

[21] *Two new convolutions for linear right-inverse operators of* d^2/dt^2, Compt. Rend. Acad. bulg. Sci. **29** (1976) 25–28.
[22] *Operational calculus for general linear differential operator of second order* (Dimovski I.; Bozhinov N.), Compt. Rend. Acad. bulg. Sci. **28**, 6 (1975) 727–730.
[23] *Boundary value operational calculi for linear differential operators of second order* (Dimovski I., Bozhinov N.), Compt. Rend. Acad. bulg. Sci. **31**, 7 (1978).
[24] *An explicit expression for the convolution of the Meijer transformation*, Compt. rend. Acad. bulg. Sci **26**, 10 (1973).
[25] Ex*plicit convolution for Hermite transform* (Dimovski I., Kalla S. L.), Math. Japonica **33**, 3 (1988) 345–351.
[26] *Finite integral transforms for non-local boundary value problems* (Dimovski I, Petrova R), in: Generalized Functions and Convergence. Memorial Volume to Professor Jan Mikusinski (Eds. P. Antosik et al.), World Scientific (1990) 89–103.
[27] *Finite integral transforms of the third kind for nonlocal boundary value problems* (Dimovski I., Petrova R.), in: "Transform Methods and Special Functions, Sofia' 94", SCT Publ., Singapore (1995) 18–32.

4. Convolutions and Integral Transforms, Related to the Hyper-Bessel Operators

[28] *Operational calculus for a class of differential operators*, Compt. rend. Acad. bulg. Sci. **19**, 12 (1966) 1111–1114.
[29] *On an operational calculus for a differential operator*, Compt. rend. Acad. bulg. Sci. **21**, 2 (1968) 513–516.

[30] *On the uniqueness of the operational field of operator*, Compt. rend. Acad. bulg. Sci. **21**, (1968) 1255–1258.

[31] *Foundations of operational calculi for Bessel-type differential operators*, Serdica. Bulg. Math. Publ. **1**, 1 (1975) 51–63.

[32] *On a Bessel-type integral transformation, due to N. Obrechkoff*, Compt. rend. Acad. bulg. Sci **27**, 1 (1974) 23–26.

[33] *A transform approach to operational calculus for the general Bessel-type differential operator*, Compt. rend. Acad. bulg. Sci. **27**, 2 (1974) 155–158.

[34] *Complex inversion formulas for the Obrechkoff transform* (Dimovski I., Kiryakova V.), Pliska **4** (1981) 110–116.

[35] *Relation between the integral transforms of Laplace and Obrechkoff* (Dimovski, I., Kiryakova, V.), in: "Generalized Functions Appl. Math. Phys., Proc. Internat. Conf. Moscow '1980" (1981) 225–231.

[36] *Obrechkoff's generalization of the Laplace and Meijer transforms: Origins and recent developments* (Dimovski I., Kiryakova V.), in: "Transform Methods and Special Functions, Varna '96. Proc. Int. Workshop", Sofia (1999) 557–576.

[37] *On an integral transformation due to N. Obrechkoff* (Dimovski I., Kiryakova V.), in: Proc. Conf. on Complex Analysis, Kozubnik '1980 = Lecture Notes in Mathematics **798** (1980) 141–147.

[38] *Transmutations, convolutions and fractional powers of Bessel-type operators via Meijer's G-function* (Dimovski I., Kiryakova V.), in: Complex Analysis and Applications, Varna '83 (1985) 45–66.

[39] *The Obrechkoff integral transform: Properties and relation to a generalized fractional calculus* (Dimovski, I, Kiryakova, V.), Numer. Funct. Anal. and Optimiz. **21**, 1–2 (2000) 121–144.

[40] *Generalized Poisson transmutations and corresponding representations of hyper-Bessel functions* (Dimovski I., Kiryakova V.), Compt. rend. Acad. bulg. Sci. **39**, 10 (1986) 29–32.

[41] *Generalized Poisson representations of hypergeometric functions* ${}_pF_q$, $p < q$ *using fractional integrals* (Dimovski I., Kiryakova V.), in: "Mathematics and Math. Educ., 16th Spring Conf. UBM" (1987) 205–212.

[42] *Isomorphism of the quotient fields, generated by Bessel-type differential operators*, Math. Nachr. **67** (1975) 101–107.

[43] *Commutants of initial right inverses of hyper-Bessel differential operators*, in: *Complex Analysis and Generalized Functions, Varna '91* (1993) 56–62.

5. Some More Recent Papers

[44] *Operational calculus approach to PDE arising in QR-regularization of ill-posed problems* (Dimovski I., Kalla S., Ali I.), Mathematical and Computer Modelling **35** (2002) 835–848.

[45] *Commutants of the Pommier operator* (Dimovski I., Hristov V.). Internat. J. of Math. and Math. Sci. **2005**, 8 (2005) 1239–1251.

[46] *Commutants of the Euler operator and corresponding mean-periodic functions* (Dimovski, I., Hristov, V.), Preprint No 2, Inst. Math. Inf.-B.A.S. (2005).

[47] *Commutants of the Dunkl operators in* $C(\mathbb{R})$ (Dimovski I., Hristov V., Sifi M.), Fract. Calc. Appl. Anal. **9**, 3 (2006) 195–213; at http://www.math.bas.bg/∼fcaa.

[48] *Mean-periodic solutions of Euler differential equations* (Dimovski I., Hristov V., Sifi M.), in: Proc. 16-th Colloq. of Tunisian Math. Soc., Sousse, March 2008.

[49] *Commutants of the square of the differentiation in* $C(R_0^+)$ (Dimovski I., Hristov V.), in: Proc. 16-th Colloq. of Tunisian Math. Soc., Sousse, March 2008.

[50] *Nonlocal operational calculi for Dunkl operators* (Dimovski I.H., Hristov V.Z.), SIGMA **5** (2009) Article # 030, 16 pp.

[51] *The commutant of the Riemann-Liouville operator of fractional integration*, *Fract. Calc. Appl. Anal.* **12**, 4 (2009) 443–448; at http://www.math.bas.bg/∼fcaa.

[52] *Operational calculus approach to nonlocal Cauchy problems* (Dimovski I., Spiridonova M.), Mathematics in Computer Science **4**, 2 (2010) 243–258; doi: 10.1007/s11786-010-0054-1.

[53] *Three-dimensional operational calculi for nonlocal evolution boundary value problems* (Dimovski I., Tsankov Y.), in: Proc. "Math. and Math. Educ. '2011", Conf. UBM (2011) 169–175.

[54] *Commutants of the square of differentiation on the half-line* (Dimovski I., Hristov V.), *Math. Balkanica* **26**, 1–2 (2012) 79–88.

[55] *Exact solutions of nonlocal BVPs for the multidimensional heat equations* (Dimovski I., Tsankov Y.), *Math. Balkanica* **26**, 1–2 (2012) 89–102.

[56] *Extension of Duhamel principle for linear nonlocal initial-boundary value problems* (Dimovski I., Spiridonova M.), in: Complex Anal. And Appl.' 13 (Proc. Intern. Conf., Sofia) (2013) 85–92; at http://www.math.bas.bg/complan/caa13/.

[57] *Commutant of a Cherednik type operator on the real line* (Dimovski I., Hristov V.), in: Complex Analysis and Applications '13 (Proc. of Intern. Conf., Sofia) (2-13) 77–84; at http://www.math.bas.bg/complan/caa13/.

[58] *Time-fractional Thornley's problem* (Bazhlekova E., Dimovski I.), J. of Inequalities and Special Functions **4**, 1 (2013) 21–35.

[59] *Exact solution for the fractional cable equation with nonlocal boundary conditions* (Bazhlekova E., Dimovski I.), Central European J. of Physics **11**, 10 (2013) 1304–1313; doi: 10.2478/s11534-013-0213-5.

[60] *Exact solution of two-term time-fractional Thornley's problem by operational method* (Bazhlekova E., Dimovski I.), Integr. Transf. Spec. Funct. **25**, 1 (2014) 61–74.

[61] *Operational calculi for nonlocal Cauchy problems in resonance cases* (Dimovski I., Spiridonova M.), in: Algebraic and Algorithmic Aspects of Differential and Integral Operators (Proc. 5th Intern. Meeting AADIOS 2012) = Lecture Motes in Computer Science **8372** (2014) 83–95; doi: 10.1007/978-3-642-54479-8_3.

[62] *Nonclassical convolutions and their uses*, Fract. Calc. Appl. Anal. **17**, 4 (2014) 936–944; doi: 10.2478/s13540-014-0207-z.

[63] *Operational calculus approach to explicit solving of initial and boundary value problems* (Dimovski I., Spiridonova M.), Physics of Particles and Nuclei Letters **12**, 3 (2015) 392–394; doi:10.1134/S1547477115030085.

[64] *Obrechkoff integral transform and its relation to generalized fractional calculus* (Kiryakova V., Dimovski I., Luchko Yu.), in: Proc. Intern. Conf. Fractional Differentiation and Applications 16, Novi Sad, 'July 18-20, 2016 (Eds. D. Spasic, N. Grahovac, M. Zigic, M. Rapaic, T. Atanackovic) (2016) 46–50.

Virginia Kiryakova
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
Acad. G. Bontchev str., Block 8, Sofia–1113, Bulgaria
E-mail address: virginia@diogenes.bg