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q-LAPLACE TRANSFORM FOR PRODUCT OF GENERAL CLASS OF q-POLYNOMIALS AND q-ANALOGUE OF I-FUNCTION

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Abstract. In this paper, the basic analogue of the Laplace transforms involving the product of a general class of q-polynomials along with q-analogue of Fox's H-function and q-analogue of I-function is evaluated. Limiting cases of the main outcomes are also evaluated. The paper shows a large variety of outcomes that can be achieved.

1. Introduction

In the concept of q-calculus, various notable integral transforms have the equivalent q-analogues, which includes the basic analogue of Laplace transforms [12], [1], the basic analogue of Sumudu transforms [2]-[4], the q-Wavelet transform [8], the q-Mellin transform [9], q-Mangontarum transforms [5], q-natural transforms [6], and many more, see also [7]. These q-integral transforms plays an important role for solving the q-fractional differential and integral equations; particularly see [10] and [16].

Investigation of integral transforms (including, q-transforms) image formulas with different special functions find important significance and applications in specific sub-fields of applied mathematical research. Therefore, a number of workers including, Albayrak et al. [3], Al-Omari [5], [6], Purohit and Ucar [16], Yadav and Purohit [24]-[26] etc., have studied the property, applications and evaluated a range of image formulas comprising q-special functions. Also, the degenerate Laplace transforms with the degenerate gamma function found in the papers [13, 14]. Recently, Vyas et al. [23] have found the q-Sumudu image formulas for a product of q-polynomials along with basic (or q-)analogue of Fox's H-function and I-functions. Motivated by the fascinating result, we further investigate the possibility of evaluating the q-Laplace transforms and generalized q-special functions for the family product of q-polynomials.

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We need some preliminaries for our investigation related to the q-calculus, which we add in this section.
Hahn [12] characterized the basic analogues of the Laplace transform

$$\varphi(p)=\int_0^\infty e^{-pt}f(p)\,d_qt\ (p>0), \tag{1.1}$$

by the subsequent q-integrals

$$_qL_p\{f(t)\}=\frac{1}{(1-q)}\int_0^{p^{-1}}E_q(qpt)f(t)d_qt \tag{1.2}$$

and

$$_q\mathcal{L}_p\{f(t)\}=\frac{1}{(1-q)}\int_0^\infty e_q(-pt)f(t)d_qt, \tag{1.3}$$

where $\Re(p)>0$ and the q-exponential series are as follow

$$e_q(t)=\sum_{m=0}^\infty\frac{t^m}{(q;q)_m}, \tag{1.4}$$

and

$$E_q(t)=\sum_{m=0}^\infty\frac{(-1)^m q^{m(m-1)/2}\,t^m}{(q;q)_m}. \tag{1.5}$$

Following Gasper and Rahman [11], the q-integration is given by

$$\int_0^y f(z)\,d(z;q)=y\,(1-q)\sum_{k=0}^\infty q^k f(xq^k). \tag{1.6}$$

By the use of result (1.6), operator (1.3) can be articulated as

$$\varphi(p)\equiv{}_qL_p\{f(t)\}=\frac{(q;q)_\infty}{p}\sum_{j=0}^\infty\frac{q^j f(p^{-1}q^j)}{(q;q)_j}. \tag{1.7}$$

The statement set out in (1.3) and (1.7) shall be characteristically addressed by

$$f(z)\supset_q\varphi(p)\ ,$$

wherein the function $f(z)$ is known as the main function, while the function $\varphi(p)$ is representing the q-Laplace image for the main function $f(z)$.
If β is real or else complex, and $|q|<1$, the q-analogues of shifted factorial, binomial expansion and gamma function are characterized by

$$(\beta;q)_n=\begin{cases}1, & if\ \ n=0\\ (1-\beta)\,(1-\beta\,q)\,\cdots(1-\beta\,q^{n-1}), & if\ n\in N,\end{cases} \tag{1.8}$$

$$(x-y)_\nu=x^\nu\prod_{n=0}^\infty\left[\frac{1-(y/x)q^n}{1-(y/x)q^{\nu+n}}\right], \tag{1.9}$$

and

$$\Gamma_q(\beta)=\frac{(q;q)_\infty(1-q)^{1-\beta}}{(q^\beta;q)_\infty}=\frac{(1-q)_{\beta-1}}{(1-q)^{\beta-1}}=\frac{(q;q)_{\beta-1}}{(1-q)^{\beta-1}}, \tag{1.10}$$

where $\beta\neq 0,\,-1,\,-2,\,\cdots$.

Presently, we lead by thinking back on an arrangement of q-polynomials $f_{n,N}\,(x;q)$ in expressions of a bounded complex sequence $\{S_{j,q}\}_{n=0}^{\infty}$, given as (cf. Srivastava and Agarwal [21])

$$f_{n,N}(x,q)=\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ Nj \end{bmatrix} S_{j,q}\; x^j \qquad (n=0,\,1,\,2...) \tag{1.11}$$

where N to be any positive integer.
Saxena and Kumar [17] derived a basic (or q-) analogue for the I-function in form of the Mellin-Barnes kind q-contour integral as:

$$I_{A_i,B_i}^{m_1,n_1}\left[x\,;q\,\middle|\,\begin{matrix}(a_j,\alpha_j)_{1,\,n},\,(a_{ji},\alpha_{ji})_{\,n+1,\,A_i}\\ \\ (b_j,\beta_j)_{1,\,m},\,(b_j,\beta_j)_{m+1,\,B_i}\end{matrix}\right]=\frac{1}{2\pi\,\omega}\int_C \Phi(s)\;x^s\;d_q s, \tag{1.12}$$

where

$$\Phi(s)=\frac{\prod_{j=1}^{m_1}G(q^{b_j-\beta_j s})\prod_{j=1}^{n_1}G(q^{1-a_j+\alpha_j s})\;\pi}{\sum_{i=1}^{r}\left\{\prod_{j=m_1+1}^{B_i}G(q^{1-b_{ji}+\beta_{ji}s})\prod_{j=n_1+1}^{A_i}G(q^{a_{ji}-\alpha_{ji}s})\right\}\,G(q^{1-s})\sin\pi s},$$

and

$$G(q^{\delta})=\left\{\prod_{n=0}^{\infty}(1-q^{\delta+n})\right\}^{-1}=\frac{1}{(q^{\delta};q)_{\infty}}.$$

provided $0\le m_1\le B_i$; $0\le n_1\le A_i$; $i=1,\,2,\,\cdots,\,r$; and r is finite; $\omega=\sqrt{-1}$.

Moreover $a_j,\,b_j,\,a_{ji},\,b_{ji}$ are complex and $\alpha_j,\,\beta_j,\,\alpha_{ji},\,\beta_{ji}$be real and positive numbers.
The curve of integration C run from $-i\infty$ to $+i\infty$ within such a way that every poles of $G(q^{b_j-\beta_j s})$; $1\le j\le m_1$, are to its right side, and those of $G(q^{1-a_j+\alpha_j s})$, $1\le j\le n_1$, are to its left and at least some $\varepsilon>0$ distance away from the contour C. The q-integral converge for $Re[s\log(x)-\log\sin\pi s]{<}0$, if huge value of $|s|$ on the contour, to facilitate if $|\arg x|{<}\pi$. It can be observed that other suitably indented contours parallel to the imaginary axis will replace the contour of integration C.
It is remarkable to view that for $A_1=A$; $B_1=B; r=1$, equation (1.12) yields q-extension for Fox's H-function given by Saxena et al. [20],

$$H_{A,B}^{m_1,n_1}\left[x\,;q\,\middle|\,\begin{matrix}(a,\alpha)\\ \\ (b,\beta)\end{matrix}\right]=\frac{1}{2\pi\,\omega}\int_C\;\phi(s)\;x^s\;d_q s, \tag{1.13}$$

where

$$\phi(s)=\frac{\prod_{j=1}^{m_1}G(q^{b_j-\beta_j s})\prod_{j=1}^{n_1}G(q^{1-a_j+\alpha_j s})\;\pi}{\prod_{j=m_1+1}^{B}G(q^{1-b_j+\beta_j s})\prod_{j=n_1+1}^{A}G(q^{a_j-\alpha_j s})G(q^{1-s})\sin\pi s},$$

If we put $\alpha_i=\beta_j=1,\,\forall\,i$ and j in the equation (1.13), thereupon it reduce to a q-analogue for Meijer's G-function known via Saxena et al. [20], i.e.

$$H_{A,B}^{m_1,n_1}\left[x;q\,\middle|\,\begin{matrix}(a,1)\\ \\ (b,1)\end{matrix}\right]\equiv G_{A,B}^{m,n}\left[x;q\,\middle|\,\begin{matrix}a_1,\cdots,a_A\\ \\ b_1,\cdots,b_B\end{matrix}\right]$$

$$= \frac{1}{2\pi\,\omega}\int_C \frac{\prod_{j=1}^{m_1} G(q^{b_j-s})\prod_{j=1}^{n_1} G(q^{1-a_j+s})\ \pi\, x^s}{\prod_{j=m_1+1}^{B} G(q^{1-b_j+s})\prod_{j=n_1+1}^{A} G(q^{a_j-s})G(q^{1-s})\sin\pi s} d_q s, \qquad (1.14)$$

where $0 \le m_1 \le B$, $0 \le n_1 \le A$ with $Re[s\log(x) - \log\sin\pi s] < 0$.
Also, if we take $n_1 = 0,\ m_1 = B$ in the result (1.14), we obtain the q-analogue for Mac Robert's E-function as:

$$G_{A,B}^{B,0}\left[x\,;q \,\middle|\, \begin{matrix} a_1,\cdots,a_A \\ b_1,\cdots,b_B \end{matrix}\right] = \mathrm{E_q[B;\, b_j:\ A;\, a_j}:x]. \qquad (1.15)$$

2. Main Results

In this part, we evaluate two theorems which represent q-Laplace image formulas involving the product of a family of q-polynomials along with q-analogue of generalized hypergeometric function.

Theorem 2.1 Let $\Re(\lambda) > 0$ and $\Re[s\log(x) - \log\sin\pi s] < 0$, then the q-Laplace transform for a product of q-analogue of I-function and q-polynomials family $f_{n,N}(x;q)$ as the subsequent formula:

$$_qL_p\left\{x^{\lambda-1} f_{n,N}(x,q) I_{A_i,B_i}^{m_1,n_1}\left[\rho\, x^k\,;q \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n_1},\,(a_{ji},\alpha_{ji})_{n_1+1,A_i} \\ (b_j,\beta_j)_{1,m_1},\,(b_j,\beta_j)_{m_1+1,B_i} \end{matrix}\right]\right\}$$

$$= \frac{p^{-\lambda}}{G(q)}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ N\ j \end{bmatrix} S_{j,q}p^{-j}$$

$$\times\, I_{A_i+1,B_i}^{m_1,n_1+1}\left[\frac{\rho}{p^k}\,;q \,\middle|\, \begin{matrix} (-j-\lambda,\,k),\,(a_j,\alpha_j)_{1,n_1},\,(a_{ji},\alpha_{ji})_{n_1+1,A_i} \\ (b_j,\beta_j)_{1,m_1},\,(b_j,\beta_j)_{m_1+1,B_i} \end{matrix}\right], \qquad (2.1)$$

where $0 \le m_1 \le B_i$; $0 \le n_1 \le A_i$; $i = 1, 2, \cdots, r$; r is finite, $|q| < 1$, $\{S_{j,q}\}_{n=0}^{\infty}$ be a bounded complex sequence and λ is any arbitrary.
Proof: On making use of results (1.11) and (1.12), the left hand side (say L) of the main result (2.1) becomes

$$L = {}_qL_p\left\{x^{\lambda-1}\sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ Nj \end{bmatrix} S_{j,q}x^j\,\frac{1}{2\pi\,\omega}\int_C \Phi(s)\ (\rho x^k)^s\ d_q s\right\},$$

or

$$L = \sum_{j=0}^{[n/N]}\begin{bmatrix} n \\ Nj \end{bmatrix} S_{j,q}x^j\,\frac{1}{2\pi\,\omega}\int_C \Phi(s)\ \rho^s\ {}_qL_p\left(x^{j+\lambda+ks-1}\right) d_q s.$$

Again, on using the known result, namely

$$_qL_p\{x^{\alpha-1}\} = \frac{\Gamma_q(\alpha)(1-q)^{\alpha-1}}{p^{\alpha}},$$

the above expression reduce to

$$L=\sum_{j=0}^{[n/N]}\left[\begin{array}{c} n \\ Nj \end{array}\right] S_{j,q}x^j\,\frac{1}{2\pi\,\omega}\int_C \Phi(s)\ \rho^s\ \frac{\Gamma_q(j+\lambda+ks)(1-q)^{j+\lambda+ks-1}}{p^{j+\lambda+ks}}d_qs.$$

On further simplification in above relation, we easily obtain the result (2.1). This completes the proof of theorem.
Now, for $r=1$, $A_1=A$; and $B_1=B$, the result (2.1) yields to the q-Laplace image formula comprising product of family of q-polynomials and the $H_q(.)$ function as folow:

Theorem 2.2:- If $\{S_{n,q}\}_{n=0}^{\infty}$ be a bounded complex sequence, let $m_1, n_1; A, B$ be positive integers with that $0\le m_1\le B$, $0\le n_1\le A$ and also N be a capricious positive integer. Then the following q-Laplace transform holds:

$$ {}_qL_p\left\{x^{\lambda-1}f_{n,N}(x,q)\,H_{A,B}^{m_1,n_1}\left[x^k\,;q\,\middle|\,\begin{array}{c}(a_1,\alpha_1),\,\dots,\,(a_A,\alpha_A)\\(b_1,\beta_1),\dots,(b_B,\beta_B)\end{array}\right]\right\}$$

$$=\frac{p^\lambda}{G(q)}\sum_{j=0}^{[n/N]}\left[\begin{array}{c} n \\ Nj \end{array}\right] S_{j,q}p^j$$

$$\times\, H_{A+1,B}^{m_1,n_1+1}\left[s^k\,;q\,\middle|\,\begin{array}{c}(-\lambda-j,k),(a_1,\alpha_1),\,\dots,\,(a_A,\alpha_A)\\(b_1,\beta_1),\dots,(b_B,\beta_B)\end{array}\right]\ ,\tag{2.2}$$

provided $\Re[s\log(x)-\log\sin\pi s]<0$ and $k>0$. In same way, certain results are obtained as special cases in the next section.

3. Special Cases

In this part, we discuss a few particular cases of the core result, by assigning an appropriate value to the parameters implicated in the results (2.1) and (2.2).
(i). If we put $N=1, S_{j,q}=1$, if $j=0$ and $S_{j,q}=0$, if $j\neq 0$, then $f_{n,N}(x,q)=1$ and we have the following q-Laplace image formula for the q-analogue of I-function:

$$ {}_qL_p\left\{x^{\lambda-1}I_{A_i,B_i}^{m_1,n_1}\left[\rho\,x^k\,;q\,\middle|\,\begin{array}{c}(a_j,\alpha_j)_{1,\,n},\,(a_{ji},\alpha_{ji})_{\,n+1,\,A_i}\\ \\(b_j,\beta_j)_{1,\,m},\,(b_j,\beta_j)_{m+1,\,B_i}\end{array}\right]\right\}$$

$$=\frac{\{G(q)\}^{-1}}{p^\lambda}I_{A_i+1,B_i}^{m_1,n_1+1}\left[\frac{\rho}{p^k}\,;q\,\middle|\,\begin{array}{c}(1-\lambda,\,k),\,(a_j,\alpha_j)_{1,\,n},\,(a_{ji},\alpha_{ji})_{\,n+1,\,A_i}\\ \\(b_j,\beta_j)_{1,\,m},\,(b_j,\beta_j)_{m+1,\,B_i}\end{array}\right]\ ,\ (k>0),\tag{3.1}$$

where $0\le m_1\le B_i$; $0\le n_1\le A_i$; $i=1,2,\cdots,r$; r is finite, $|q|<1$ and for arbitrary λ and ρ.
(ii). Again, for $r=1$, $A_1=A$; and $B_1=B$, the result (3.1) yields to the q-Laplace image formula comprising the $H_q(.)$ function as

$$ {}_qL_p\left\{x^{\lambda}H_{A,B}^{m_1,n_1}\left[\rho\,x^k\,;q\,\middle|\,\begin{array}{c}(a,\alpha)\\ \\(b,\beta)\end{array}\right]\right\}$$

$$= \frac{\{G(q)\}^{-1}}{p^{\lambda+1}} H^{m_1,n_1+1}_{A+1,B}\left[\frac{\rho}{p^k}\,;q\,\middle|\,\begin{array}{c}(-\lambda,\,k),\,(a,\alpha)\\ \\ (b,\beta)\end{array}\right], \tag{3.2}$$

where $0 \le m_1 \le B$; $0 \le n_1 \le A_i$; $|q| < 1$ and for arbitrary λ and ρ.
(iii). Further, if we put $\rho = 1$ and $k = 1$, the result (3.2) yields to the outcome due to Yadav and Purohit [22], namely

$$ {}_qL_p\left\{x^{\lambda} H^{m_1,n_1}_{A,B}\left[x\,;q\,\middle|\,\begin{array}{c}(a,\alpha)\\ \\ (b,\beta)\end{array}\right]\right\}$$
$$= \frac{\{G(q)\}^{-1}}{p^{\lambda+1}} H^{m_1,n_1+1}_{A+1,B}\left[\frac{1}{p^k}\,;q\,\middle|\,\begin{array}{c}(0,\,1),\,(a,\alpha)\\ \\ (b,\beta)\end{array}\right], \tag{3.3}$$

where $0 \le m_1 \le B$; $0 \le n_1 \le A$; $|q| < 1$ and for arbitrary λ.
(iv). Lastly, it is very intriguing to see that in sight of the point of limit formulae

$$\underset{q\to 1^-}{Lt}\ \frac{(q^{\alpha};q)_n}{(1-q)^n} = (\alpha)_n \text{ and } \underset{q\to 1^-}{Lt}\ \Gamma_q(\alpha) = \Gamma(\alpha),$$

the result (3.3) is a q-extension of the identified consequence due to Srivastava et al. [22], namely

$$ {}_qL_p\left\{x^{\lambda} H^{m_1,n_1}_{A,B}\left[\rho\, x^k\,\middle|\,\begin{array}{c}(a,\alpha)\\ \\ (b,\beta)\end{array}\right]\right\} = \frac{1}{p^{\lambda+1}} H^{m_1,n_1+1}_{A+1,B}\left[\frac{\rho}{p^k}\,\middle|\,\begin{array}{c}(-\lambda,\,k),\,(a,\alpha)\\ \\ (b,\beta)\end{array}\right]. \tag{3.4}$$

4. Conclusion

We conclude with the remark that the outcomes examined in this paper are common in character and the latest input to the theory of q-series. It is also significant to mention that the consequences proved in this paper may use to find some solutions of certain q-integral and q-difference equations and related to q-polynomials, the basic analogues of Meijer's G-function, Fox's H-function and I-function. Further, one can obtain a number of image formulas involving orthogonal q-polynomials as particular cases of our results, by giving special values to the sequence $S_{j,q}$, in the family of polynomials which include the polynomials viz. q-Laguerre, q-Hermite, q-Jacobi, q-Konhauser polynomials and many others.

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