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WEIGHTED NORLUND-EULER STATISTICAL CONVERGENCE IN INTUITIONISTIC FUZZY NORMED LINEAR SPACES

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Abstract. In this paper, we define weighted Norlund-Euler statistical convergence and $(N,p,q)(E,q)$-summability with respect to the intuitionistic fuzzy norm (μ, υ). We also establish relationship between these concepts and prove some theorems.

1. Introduction

Fuzzy set theory which has been defined by Zadeh [1] plays an essential role in computer programming [2], engineering [3], nonlinear dynamic systems [4], population dynamics [5] and physics [6]. Atanassov [7] introduced a new concept which is entitled as intuitionistic fuzzy set. Park [8] and Saadati and Park [9] introduced the notions of intuitionistic fuzzy metric spaces and intuitionistic fuzzy normed spaces by the means of intuitionistic fuzzy sets, respectively.

Steinhaus [10] and Fast [11] defined the notion of statistical convergence to extend the convergence of sequences as follows:

Let $\mathbb{N}$ be set of natural numbers, $K \subseteq \mathbb{N}$ and $K_n = \{k \leq n : k \in K\}$. The natural density of K is defined by $\delta(K) = \lim_{n\to\infty} n^{-1}|K_n|$ if the limit exists, where $|K_n|$ denotes the cardinality of K_n. A sequence $x = (x_k)$ is statistical convergent to L if for each $\varepsilon > 0$, the set $\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ has natural density zero, i.e. for each $\varepsilon > 0$, $\lim_{n\to\infty} \frac{1}{n}|\{j \leq n : |x_j - L| \geq \varepsilon\}| = 0$. In this case, we write $S-\lim x = L$.

Over the years, a lot of researchers discussed statistical convergence in the theory of Fourier analysis, ergodic theory and number theory. Schoenberg [27] established the relationship between the summability theory and statistical convergence. Many years later, Karakuş et al. [14] introduced the notion of statistical convergence on intuitionistic fuzzy normed linear spaces. Extending this idea, various investigations [19],[32] and [33] have been done. Recently, Altundağ and Kamber [15],[16], Kamber [17] and Savaş et al. [29]-[31] studied the relationship between various statistical

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convergence types and certain summability methods in intuitionistic fuzzy normed linear spaces, respectively.

In this article, we will introduce weighted Norlund-Euler statistical convergence and $(N,p,q)\,(E,q)$-summability with respect to the intuitionistic fuzzy norm (μ,υ).

2. Basic definitions

In this section, we give some definitions and notations which will be used for this study.

Definition 2.1. ([18]) Let $\sum\limits_{k=0}^{\infty} x_k$ be a given infinite series with sequence of its nth partial sum (S_n). Let (E,q) transform is defined as

$$E_n^{(E,q)} = \frac{1}{(1+q)^n}\sum_{k=0}^{n}\binom{n}{k}q^{n-k}S_k.$$

If $\lim\limits_{n\to\infty} E_n^{(E,q)} = S$, then the series $\sum\limits_{k=0}^{\infty} x_k$ is (E,q)-summable to a definite number S. In this case, it is denoted by $S_n \to S(E,q)$ as $n\to\infty$. Let (p_n) and (q_n) be two sequences of non-zero real constants such that

$$P_n = \sum_{k=0}^{n} p_k, P_{-1} = p_{-1} = 0,$$

and

$$Q_n = \sum_{k=0}^{n} q_k, Q_{-1} = q_{-1} = 0.$$

For given sequences (p_n) and (q_n), convolution $p*q$ is defined by

$$R_n = p*q = \sum_{k=0}^{n} p_k q_{n-k}.$$

Let (N,p,q) transform is defined as

$$t_n^{(N,p,q)} = \frac{1}{R_n}\sum_{v=0}^{n} p_{n-v}q_v S_v.$$

If $\lim\limits_{n\to\infty} t_n^{(N,p,q)} = S$, the series $\sum\limits_{k=0}^{\infty} x_k$ or the sequence (S_n) is summable to S by generalized Norlund method (or (N,p,q)-summable to S). It is denoted by $S_n \to S(N,p,q)$ as $n\to\infty$.

Let us define following sequence:

$$t_n^{(N,p,q)(E,q)} = \tfrac{1}{R_n}\sum_{k=0}^{n} p_{n-k}q_k E_k^q = \tfrac{1}{R_n}\sum_{k=0}^{n} p_{n-k}q_k \tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}S_v.$$

If $\lim\limits_{n\to\infty} t_n^{(N,p,q)(E,q)} = S$, then the series $\sum\limits_{k=0}^{\infty} x_k$ or the sequence (S_n) is summable to S by Norlund-Euler method (or $(N,p,q)(E,q)$-summable) and it is denoted by $S_n \to S(N,p,q)(E,q)$ as $n\to\infty$. When we take $p_k = 1, q_k = 1$ for all $k\in\mathbb{N}$, then Euler summability method is obtained.

Definition 2.2. ([18]) Let $K \subseteq \mathbb{N}$. The number

$$\delta_{NE}^{q}(K) = \lim_{n\to\infty} \frac{1}{R_n} \left|\{k \le R_n : k \in K\}\right|$$

is said to be weighted Norlund-Euler density of K. A sequence $x = (x_k)$ is said to be generalized weighted Norlund-Euler statistically convergent (or S_{NE}^{q} -convergent) to L if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{R_n} \left| \left\{ k \le R_n : p_{n-k} q_k \frac{1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \left|x_v - L\right| \ge \varepsilon \right\} \right| = 0.$$

In this case, we write $S_{NE}^{q} - \lim x = L$.

Definition 2.3. ([20]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0,1]$,
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for each $a,\ b,\ c, d \in [0,1]$.

Definition 2.4. ([20]) A binary operation $\circ : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

(i) $\circ$ is associative and commutative,
(ii) $\circ$ is continuous,
(iii) $a \circ 0 = a$ for all $a \in [0,1]$,
(iv) $a \circ b \le c \circ d$ whenever $a \le c$ and $b \le d$ for each $a,\ b,\ c, d \in [0,1]$.

Definition 2.5. ([21]) The five-tuple $(X,\ \mu, \upsilon, *, \circ)$ is said to be intuitionistic fuzzy normed linear space (or shortly IFNLS) where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, $\mu,\ \upsilon$ are fuzzy sets on $X \times (0,\infty)$, μ denotes the degree of membership and υ denotes the degree of nonmembership of $(x,t) \in X \times (0,\infty)$ satisfying the following conditions for every $x, y \in X$ and $s, t > 0$:

(i) $\mu(x,t) + \upsilon(x,t) \le 1$,
(ii) $\mu(x,t) > 0$,
(iii) $\mu(x,t) = 1$ if and only if $x = 0$,
(iv) $\mu(\alpha x, t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \ne 0$,
(v) $\mu(x,t) * \mu(y,s) \le \mu(x+y, t+s)$,
(vi) $\mu(x,.) : (0,\infty) \to [0,1]$ is continuous,
(vii) $\lim_{t\to\infty} \mu(x,t) = 1$ and $\lim_{t\to 0} \mu(x,t) = 0$,
(viii) $\upsilon(x,t) < 1$,
(ix) $\upsilon(x,t) = 0$ if and only if $x = 0$,
(x) $\upsilon(\alpha x, t) = \upsilon\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \ne 0$,
(xi) $\upsilon(x,t) \circ \upsilon(y,s) \ge \upsilon(x+y, s+t)$,
(xii) $\upsilon(x,.) : (0,\infty) \to [0,1]$ is continuous,
(xiii) $\lim_{t\to\infty} \upsilon(x,t) = 0$ and $\lim_{t\to 0} \upsilon(x,t) = 1$.

In this case (μ, υ) is called intuitionistic fuzzy linear norm.

Example 2.1. ([21]) Let(X, $\|.\|$) be a normed linear space, and let $a * b = ab$ and $a \circ b = \min\{a + b, 1\}$ for all $a, b \in [0, 1]$. For all $x \in X$ and every $t > 0$, consider

$$\mu(x,t) := \tfrac{t}{t+\|x\|} \text{ and } \upsilon(x,t) := \tfrac{\|x\|}{t+\|x\|}.$$

Then $(X, \mu, \upsilon, *, \circ)$ is an IFNLS.

Definition 2.6. ([21]) Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is convergent to $L \in X$ with respect to the intuitionistic fuzzy linear norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $\mu(x_k - L, t) > 1 - \varepsilon$ and $\upsilon(x_k - L, t) < \varepsilon$ for all $k \geq k_0$ where $k \in \mathbb{N}$. It is denoted by $(\mu, \upsilon) - \lim x = L$.

Theorem 2.1. *([22])Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. Then, a sequence $x = (x_k)$ in X is convergent to $L \in X$ if and only if $\lim_{k\to\infty} \mu(x_k - L, t) = 1$ and $\lim_{k\to\infty} \upsilon(x_k - L, t) = 0$.*

3. Main results

In this section, we define a new summability method named as $(N, p, q)(E, q)^{(\mu,\upsilon)}$-summability. Benefiting from this definition, we define a new concept of statistical convergence named as weighted Norlund-Euler statistical convergence with respect to the intuitionistic fuzzy norm (μ, υ). Moreover, we investigate some connections between these concepts.

Definition 3.1. Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is said to be $(N, p, q)(E, q)$-summable to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, υ) (or $(N, p, q)(E, q)^{(\mu,\upsilon)}$- summable to $L \in X$) if, for every $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\frac{1}{R_n}\sum_{k=0}^{n} \mu\left(p_{n-k}q_k \frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v} q^{k-v}(x_v - L), t\right) > 1 - \varepsilon$$

and

$$\frac{1}{R_n}\sum_{k=0}^{n} \upsilon\left(p_{n-k}q_k \frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v} q^{k-v}(x_v - L), t\right) < \varepsilon$$

for all $n \geq n_0$. In this case, we write $(N, p, q)(E, q)^{(\mu,\upsilon)} - \lim x = L$.

Definition 3.2. Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is said to be weighted Norlund-Euler statistical convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ,υ) (or $S_{NE}^{q(\mu,\upsilon)}$- convergent to $L\in X$) if, for every $\varepsilon>0$ and $t>0$,

$$\lim_{n\to\infty}\frac{1}{R_n}\left|\left\{k\le R_n : \mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\le 1-\varepsilon\right.\right.$$
$$\left.\left.\text{or }\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\ge\varepsilon\right\}\right|=0.$$

In this case, we write $S_{NE}^{q(\mu,\upsilon)}-\lim x=L$.

In the following theorems, we establish the relation between weighted Norlund Euler statistical convergence and $(N,p,q)\,(E,q)$- summability in intuitionistic fuzzy normed linear spaces.

Theorem 3.1. *Let $(X,\,\mu,\upsilon,*,\circ)$ be an IFNLS, $x=(x_k)$ be a sequence in X and $\frac{R_n}{n}\ge 1$ for all $n\in\mathbb{N}$. If $S_{NE}^{q(\mu,\upsilon)}-\lim x=L.$, $\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\ge 1-M$ and $\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\le M$ for all $k\in\mathbb{N}$, then $(N,p,q)\,(E,q)^{(\mu,\upsilon)}-\lim x=L$.*

Proof. Suppose that $S_{NE}^{q(\mu,\upsilon)}-\lim x=L$. Then, for every $\varepsilon>0$ and $t>0$, we define the sets $K_{R_n}(\varepsilon)$ and $K_{R_n}^c(\varepsilon)$ such that

$$K_{R_n}(\varepsilon)=\left\{k\le R_n : \mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\le 1-\varepsilon\right.$$
$$\left.\text{or }\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\ge\varepsilon\right\}$$

and

$$K_{R_n}^c(\varepsilon)=\left\{k\le R_n : \mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)> 1-\varepsilon\right.$$
$$\left.\text{and }\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)<\varepsilon\right\}.$$

Hence

$$\frac{1}{R_n}\sum_{k=0}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)$$
$$=\frac{1}{R_n}\sum_{\substack{k=0\\k\in K_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)+$$

$$+\frac{1}{R_n}\sum_{\substack{k=0\\ k\in K^c_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)$$
$$=S_1(n)+S_2(n)$$

where

$$S_1(n)=\frac{1}{R_n}\sum_{\substack{k=0\\ k\in K_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right) \tag{3.1}$$

and

$$S_2(n)=\frac{1}{R_n}\sum_{\substack{k=0\\ k\in K^c_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right). \tag{3.2}$$

If $k\in K_{R_n}(\varepsilon)$, then

$$S_1(n)=\frac{1}{R_n}\sum_{\substack{k=0\\ k\in K_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\geq\frac{|K_{R_n}(\varepsilon)|}{R_n}(1-M). \tag{3.3}$$

Since $S^{q(\mu,\upsilon)}_{NE}-\lim x=L$.

$$\lim_{n\to\infty}S_1(n)\geq 0. \tag{3.4}$$

If $k\in K^c_{R_n}(\varepsilon)$, then we have

$$S_2(n)=\frac{1}{R_n}\sum_{\substack{k=0\\ k\in K^c_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)>\frac{\left|K^c_{R_n}(\varepsilon)\right|}{R_n}(1-\varepsilon) \tag{3.5}$$

which yields that

$$\lim_{n\to\infty}S_2(n)>(1-\varepsilon). \tag{3.6}$$

Using equalities (3.1)-(3.2) and inequalities (3.3)-(3.6), we get

$$\lim_{n\to\infty}\frac{1}{R_n}\sum_{k=0}^{n}\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)=1. \tag{3.7}$$

Similarly, we obtain

$$\lim_{n\to\infty}\frac{1}{R_n}\sum_{k=0}^{n}\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)=0. \tag{3.8}$$

As a result of equalities (3.8) and (3.9), we have

$$(N,p,q)(E,q)^{(\mu,\upsilon)}-\lim x=L. \tag{3.9}$$

Theorem 3.2. *Let $(X,\mu,\upsilon,*,\circ)$ be an IFNLS and $\frac{R_n}{n}\geq 1$ for all $n\in\mathbb{N}$. If a sequence $x=(x_k)$ in X is $(N,p,q)(E,q)^{(\mu,\upsilon)}$- summable to $L\in X$, then a sequence $x=(x_k)$ is $S^{q(\mu,\upsilon)}_{NE}$- convergent to $L\in X$.*

Proof. Suppose that $x=(x_k)$ is $(N,p,q)\,(E,q)^{(\mu,\upsilon)}$-summable to $L\in X$. For every $\varepsilon>0$ and $t>0$, define the sets $K_{R_n}(\varepsilon)$ and $K^c_{R_n}(\varepsilon)$ such that

$$K_{R_n}(\varepsilon)=\left\{k\le R_n:\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\le 1-\varepsilon\right.$$
$$\left.\text{or }\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)\ge\varepsilon\right\}$$

and

$$K^c_{R_n}(\varepsilon)=\left\{k\le R_n:\mu\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)> 1-\varepsilon\right.$$
$$\left.\text{and }\upsilon\left(p_{n-k}q_k\frac{1}{(1+q)^k}\sum_{v=0}^{k}\binom{k}{v}q^{k-v}(x_v-L),t\right)<\varepsilon\right\}.$$

Then

$$\begin{aligned}
&\tfrac{1}{R_n}\sum_{k=0}^{n}\mu\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)\\
&=\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)+\\
&+\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K^c_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)\\
&\ge\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K^c_{R_n}(\varepsilon)}}^{n}\mu\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)>\tfrac{1}{R_n}\left|K^c_{R_n}(\varepsilon)\right|(1-\varepsilon)\,.
\end{aligned}\tag{3.10}$$

As a consequence of inequality (3.10), we get $\lim_{n\to\infty}\frac{1}{R_n}\left|K^c_{R_n}(\varepsilon)\right|=1$. Similarly, for every $\varepsilon>0$ and $t>0$,

$$\begin{aligned}
&\tfrac{1}{R_n}\sum_{k=0}^{n}\upsilon\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)\\
&=\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K_{R_n}(\varepsilon)}}^{n}\upsilon\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)+\\
&+\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K^c_{R_n}(\varepsilon)}}^{n}\upsilon\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)\\
&\ge\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K_{R_n}(\varepsilon)}}^{n}\upsilon\left(p_{n-k}q_k\tfrac{1}{(1+q)^k}\sum_{v=0}^{k}\tbinom{k}{v}q^{k-v}(x_v-L),t\right)\ge\tfrac{1}{R_n}\sum_{\substack{k=0\\k\in K_{R_n}(\varepsilon)}}^{n}\varepsilon=\tfrac{1}{R_n}\left|K_{R_n}(\varepsilon)\right|\varepsilon.
\end{aligned}\tag{3.11}$$

By inequality (3.11), we have $\lim_{n\to\infty}\frac{1}{R_n}\left|K_{R_n}(\varepsilon)\right|=0$. This completes the proof.

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