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THE NEUTRIX LIMIT OF A k-DEFORMATION OF THE GAMMA FUNCTION

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Abstract. The main object of this paper is to give a definition of k-deformation of the gamma function for all real values of x.

1. Introduction and Preliminaries

One of the most important special functions is the Euler's Gamma function. It has many applications in various fields like mathematical physics, analysis and statistics, and defined for $x > 0$ by the Riemann integral

$$\Gamma(x) = \int_0^\infty t^{x-1}\, e^{-t}\, dt,$$

[1, 3, 12].

Let N' be a nonempty set and let $\mathcal{N}$ be a commutative, additive group of functions mapping from N' to a commutative, additive group N''. The group $\mathcal{N}$ is called "neutrix" if the function which is identically equal to zero is the only constant function occurring in $\mathcal{N}$ and the functions in the set $\mathcal{N}$ are called "negligible". Now let $N^{'}$ be a domain lying in a topological space with a limit point b not belonging to $N^{'}$ and $\mathcal{N}$ be a commutative additive group of functions defined on N' with the property that "$f \in \mathcal{N}$, $\lim\limits_{\varepsilon\to b} f(\varepsilon) = c\,(\text{constant})$ for $\varepsilon \in N^{'}$ then $c = 0$". If it is possible to find a constant c such that $f(\varepsilon) - c$ is negligible, then, c is called the neutrix limit of $f(\varepsilon)$ as ε tends to b and denoted by $\mathrm{N}-\lim\limits_{\varepsilon\to b} f(\varepsilon) = c$. Note that more information about neutrix calculus can be found in [2].

Using the regularization technique given by Gel'fand and Shilov the gamma function is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1}\Big[e^{-t} - \sum_{i=0}^{n-1}(-1)^i \frac{t^i}{i!}\Big]\, dt$$

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for $-n < x < -n+1$, $n = 1, 2, \ldots$, [7], and using the concept of neutrix and neutrix limit; the gamma function is defined for all real values of x such that,

$$\Gamma(x) = \mathrm{N-}\lim_{\epsilon\to 0} \int_{\epsilon}^{\infty} t^{x-1}\, e^{-t}\, dt, \tag{1.1}$$

[6].

Recently, Diaz and Pariguan in [4, 5] introduced a k-deformation of the gamma function as

$$\Gamma_k(x) = \int_0^{\infty} t^{x-1} e^{-\frac{t^k}{k}}\, dt \tag{1.2}$$

and many researchers gave some properties and inequalities for the above function [8, 9, 10, 11]. Note that the k-deformation of the gamma function or simply k-gamma function $\Gamma_k(x)$ satisfies the following properties: $\Gamma_k(x+k) = x\Gamma_k(x)$, $\Gamma_k(k) = 1$ and $\Gamma_k(x) \to \Gamma(x)$ as $k \to 1$.

2. Main Results

In this present paper, we are going to use the regularization technique and the concepts of neutrix and neutrix limit to give a generalization of $\Gamma_k(x)$ for all real values of x.

Definition 2.1. *For $k > 0$ and all real values of x, the k-deformation of the gamma function is defined by*

$$\Gamma_k(x) = \mathrm{N-}\lim_{\epsilon\to 0} \int_{\epsilon}^{\infty} t^{x-1}\, e^{-\frac{t^k}{k}}\, dt, \tag{2.1}$$

where $\mathcal{N}$ is the neutrix having domain $N^{'} = (0, \infty)$, range $N^{''} = \mathbb{R}$ with negligible functions of finite linear sums of the functions

$$\epsilon^{\lambda} \ln^{r-1} \epsilon, \ln^r \epsilon \qquad (\lambda < 0, r = 1, 2, \ldots)$$

and all functions $f(\epsilon)$ which converge to zero in the normal sense as ϵ tends to zero.

It is not immediately obvious that the neutrix limit in the equation (2.1) exists. We will prove that this neutrix limit exists so that $\Gamma_k(x)$ is well-defined for the given values of k and x in the Definition 2.1.

Theorem 2.2. *The neutrix limit as ϵ tends to zero of the Riemann integral*

$$\int_{\epsilon}^{\infty} t^{x-1} e^{-\frac{t^k}{k}}\, dt$$

exists for all real values of x and $k > 0$.

Proof. We need to find out the existence of the neutrix limit of the integral not only for x but also we have to look for different values of k. So we try to obtain the result by following steps.

- **Case 1.** $k > 0$ and $x > 0$,
- **Case 2.** $k > 1$,
  (1) $-k + 1 < x < 0$
  (2) $-(n+1)k + 1 < x < -nk + 1$ and $x \neq -nk$ for $n = 1, 2, \ldots$
  (3) $x = -nk$ for $n = 1, 2, \ldots$
- **Case 3.** $\frac{1}{m+1} < k < \frac{1}{m}$ for $m = 1, 2, \ldots,$

(1) $-(m+1)k+1 \leq x < 0$
(2) $-(n+1)k+1 \leq x < -nk+1$ for $n = m+1, m+2, \dots$, $x \neq -(n-m)k$
(3) $x = -(n-m)k$ for $n = m+1, m+2, \dots$

- **Case 4.** $k = \frac{1}{l}$, $l = 2, 3, \dots$, $-(n+1)k+1 \leq x < -nk+1$, $n = l, l+1, l+2 \dots$,
- **Case 5.** $k > 0$ such that $\frac{n+1}{m+1} \leq k < \frac{n+1}{m}$ for $m = 1, 2, \dots$,
  (1) $j \in \{1, 2, \dots, m\}$ such that $kj = n$
  (2) $j \notin \{1, 2, \dots, m\}$ such that $kj = n$.

Hence by the aid of these cases, we will able to expand the domain of the integral for all real values of x and $k > 0$.

Case 1: Let $k > 0$ and $x > 0$. Then the integral

$$\int_0^\infty t^{x-1} e^{-\frac{t^k}{k}}\, dt$$

is convergent. Hence

$$\underset{\epsilon\to 0}{\mathrm{N-lim}} \int_\epsilon^\infty t^{x-1}\, e^{-\frac{t^k}{k}}\, dt = \lim_{\epsilon\to 0} \int_\epsilon^\infty t^{x-1}\, e^{-\frac{t^k}{k}}\, dt = \int_0^\infty t^{x-1}\, e^{-\frac{t^k}{k}}\, dt.$$

Case 2: Let $k > 1$ and $-k+1 \leq x < 0$. Then

$$\begin{aligned} &\underset{\epsilon\to 0}{\mathrm{N-lim}} \int_\epsilon^\infty t^{x-1}\, e^{-\frac{t^k}{k}}\, dt = \\ &= \underset{\epsilon\to 0}{\mathrm{N-lim}} \int_\epsilon^1 t^{x-1}\, [e^{-\frac{t^k}{k}} - 1]\, dt + \underset{\epsilon\to 0}{\mathrm{N-lim}} \int_\epsilon^1 t^{x-1}\, dt + \int_1^\infty t^{x-1} e^{-\frac{t^k}{k}}\, dt = \\ &= \int_0^\infty t^{x-1}\, [e^{-\frac{t^k}{k}} - H(1-t)]\, dt + \frac{1}{x}. \end{aligned} \tag{2.2}$$

where $H(t)$ is Heaviside function.

Now let $k > 1$ and $-(n+1)k+1 \leq x < -nk+1$ and $x \neq -nk$ for $n = 1, 2, \dots$. Then we get,

$$\begin{aligned} &\underset{\epsilon\to 0}{\mathrm{N-lim}} \int_\epsilon^\infty t^{x-1}\, e^{-\frac{t^k}{k}}\, dt = \underset{\epsilon\to 0}{\mathrm{N-lim}} \int_\epsilon^1 t^{x-1} \Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n} (-1)^j \frac{t^{kj}}{k^j j!}\Big]\, dt + \\ &+ \int_1^\infty t^{x-1} e^{-\frac{t^k}{k}}\, dt + \sum_{j=0}^{n} \frac{(-1)^j}{k^j j!}\, \underset{\epsilon\to 0}{\mathrm{N-lim}} \frac{t^{x+kj}}{x+kj}\Big|_\epsilon^1 = \\ &= \int_0^1 t^{x-1} \Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n} (-1)^j \frac{t^{kj}}{k^j j!}\Big]\, dt + \int_1^\infty t^{x-1} e^{-\frac{t^k}{k}}\, dt + \sum_{j=0}^{n} \frac{(-1)^j}{k^j j! (x+kj)}. \end{aligned}$$

Hence we can write

$$\Gamma_k(x) = \int_0^\infty t^{x-1} \Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n} (-1)^j \frac{t^{kj}}{k^j j!}\, H(1-t)\Big]\, dt + \sum_{j=0}^{n} \frac{(-1)^j}{k^j j! (x+kj)} \tag{2.3}$$

for $k > 1$, $-(n+1)k+1 \leq x < -nk+1$ and $x \neq -nk$, $n = 1, 2, \dots$, $k > 1$.

Now, we need to look for $x = -nk$ for $n = 1, 2, \dots$. For simplicity let $x = -k$ and

$k > 1$. Then we will define $\Gamma_k(-k)$:

$$\begin{aligned} &\mathrm{N-}\lim_{\epsilon\to 0}\int_\epsilon^\infty t^{-k-1}\,e^{-\frac{t^k}{k}}\,dt = \\ &= \mathrm{N-}\lim_{\epsilon\to 0}\int_\epsilon^1 t^{-k-1}\,[e^{-\frac{t^k}{k}} - 1 + \frac{t^k}{k}]\,dt + \\ &+ \int_1^\infty t^{-k-1}e^{-\frac{t^k}{k}}\,dt + \mathrm{N-}\lim_{\epsilon\to 0}\int_\epsilon^1 t^{-k-1}\,dt - \mathrm{N-}\lim_{\epsilon\to 0}\frac{1}{k}\int_\epsilon^1 t^{-1}\,dt. \end{aligned}$$

Since $\ln\epsilon$ is a negligible function, we get

$$\Gamma_k(-k) = \int_0^\infty t^{-k-1}\,[e^{-\frac{t^k}{k}} - (1 - \frac{t^k}{k})\,H(1-t)]\,dt - \frac{1}{k} \tag{2.4}$$

for $k > 1$. And more generally we have

$$\begin{aligned} &\Gamma_k(x) = \mathrm{N-}\lim_{\epsilon\to 0}\int_\epsilon^\infty t^{x-1}\,e^{-\frac{t^k}{k}}\,dt = \\ &= \int_0^\infty t^{x-1}\Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n}(-1)^j\frac{t^{kj}}{k^j j!}\,H(1-t)\Big]\,dt + \sum_{j=0}^{n-1}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj+x} \end{aligned} \tag{2.5}$$

for $k > 1$ and $x = -nk$, $n = 1, 2, \ldots$.
Up to now, we have obtained the result only for $k > 1$.
Case 3: Now, let $\frac{1}{m+1} < k < \frac{1}{m}$, $m = 1, 2, \ldots$. Then for $-(m+1)k + 1 \le x < 0$ we have

$$\begin{aligned} &\mathrm{N-}\lim_{\epsilon\to 0}\int_\epsilon^\infty t^{x-1}\,e^{-\frac{t^k}{k}}\,dt = \\ &= \int_0^\infty t^{x-1}\Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{m}(-1)^j\frac{t^{kj}}{k^j j!}H(1-t)\Big]\,dt + \sum_{j=0}^{m}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj+x}. \end{aligned}$$

Here we don't need to omit the value $x = -mk$, since $-mk \notin [-(m+1)k+1, 0)$.
Now, let $\frac{1}{m+1} < k < \frac{1}{m}$, $-(n+1)k + 1 \le x < -nk + 1$ for $n = m+1, m+2, \ldots$, $x \ne -(n-m)k$, $m = 1, 2, \ldots$. Then

$$\begin{aligned} &\mathrm{N-}\lim_{\epsilon\to 0}\int_\epsilon^\infty t^{x-1}\,e^{-\frac{t^k}{k}}\,dt = \\ &= \int_0^\infty t^{x-1}\Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n}(-1)^j\frac{t^{kj}}{k^j j!}H(1-t)\Big]\,dt + \sum_{j=0}^{n}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj+x}. \end{aligned} \tag{2.6}$$

Now, let $\frac{1}{m+1} < k < \frac{1}{m}$, $m = 1, 2, \ldots$ and $x = -(n-m)k$ for $n = m+1, m+2, \ldots$.
Note that, $x = -(n-m)k \in [-(n+1)k+1, -nk+1)$. Hence we can write

$$\begin{aligned}
&\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_{\epsilon}^{\infty} t^{-(n-m)k-1}\, e^{-\frac{t^k}{k}}\, dt = \\
&= \int_0^{\infty} t^{-(n-m)k-1}\Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n}(-1)^j\,\frac{t^{kj}}{k^j j!}H(1-t)\Big]\, dt + \\
&+ \sum_{j=0}^{n-m-1}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj-(n-m)k} + \sum_{j=n-m+1}^{n}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj-(n-m)k}. \qquad (2.7)
\end{aligned}$$

Since $x=-(n-m)k$ for $n=m+1,m+2,\ldots$, then we have $x=-k$, $x=-2k$, $x=-3k,\ldots$. Hence we got that $\Gamma_k(-nk)$ for $n=1,2,\ldots$ and $\frac{1}{m+1}<k<\frac{1}{m}$, $m=1,2,\ldots$.

Case 4: Let $k=\frac{1}{l}$, $l=2,3,\ldots$, $-(n+1)k+1\le x<-nk+1$, $n=l,l+1,l+2\ldots$. Note that, if $n=l$, then $-nk+1=0$, so that $-(n+1)k+1\le x<0$. Hence, we need to get the result for the values $k=\frac{1}{l}$ separately. Let $x\neq-[(n-l)+1]k$, then

$$\begin{aligned}
&\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_{\epsilon}^{\infty} t^{x-1}\, e^{-\frac{t^k}{k}}\, dt = \\
&= \int_0^{\infty} t^{x-1}\Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n}(-1)^j\,\frac{t^{kj}}{k^j j!}\,H(1-t)\Big]\, dt + \sum_{j=0}^{n}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj+x}. \qquad (2.8)
\end{aligned}$$

and for $x=-[(n-l)+1]k$, we have

$$\begin{aligned}
&\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_{\epsilon}^{\infty} t^{x-1}\, e^{-\frac{t^k}{k}}\, dt = \\
&= \int_0^{\infty} t^{x-1}\Big[e^{-\frac{t^k}{k}} - \sum_{j=0}^{n}(-1)^j\,\frac{t^{kj}}{k^j j!}\,H(1-t)\Big]\, dt + \\
&+ \sum_{j=0}^{n-l}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj+x} + \sum_{j=n-l+2}^{n}\frac{(-1)^j}{k^j j!}\,\frac{1}{kj+x}. \qquad (2.9)
\end{aligned}$$

Now we will prove the existence of $\Gamma_k(-n)$ for $n=0,1,2,\ldots$. For $k\ge n+1$, one can see that

$$\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_{\epsilon}^{\infty} t^{-n-1}\, e^{-\frac{t^k}{k}}\, dt = \int_0^{\infty} t^{-n-1}\Big[e^{-\frac{t^k}{k}} - H(1-t)\Big]\, dt. \qquad (2.10)$$

Case 5: Now, let the number $k>0$ be such that $\frac{n+1}{m+1}\le k<\frac{n+1}{m}$ for $m=1,2,\ldots$. If there exist a number j in the set $\{1,2,\ldots,m\}$ satisfies the condition that $kj=n$, we have

$$
\begin{aligned}
\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_\epsilon^\infty t^{-n-1}\,e^{-\frac{t^k}{k}}\,dt &= \mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_\epsilon^1 t^{-n-1}\Big[e^{-\frac{t^k}{k}}-\sum_{j=0}^{m}(-1)^j\frac{t^{kj}}{k^j j!}\Big]\,dt+\\
&+\int_1^\infty t^{-n-1}e^{-\frac{t^k}{k}}\,dt+\sum_{j=0}^{m}\frac{(-1)^j}{k^j j!}\,\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_\epsilon^1 t^{kj-n-1}\,dt=\\
&=\int_0^\infty t^{-n-1}\Big[e^{-\frac{t^k}{k}}-\sum_{j=0}^{m}(-1)^j\frac{t^{kj}}{k^j j!}H(1-t)\Big]\,dt+\sum_{j=0,kj\neq n}^{m}\frac{(-1)^j}{k^j j!}\frac{1}{kj-n}+\\
&+\mathrm{N}\!-\!\lim_{\epsilon\to 0}\frac{(-1)^{\frac{n}{k}}}{k^{\frac{n}{k}}(\frac{n}{k})!}\int_\epsilon^1 t^{k(\frac{n}{k})-n-1}\,dt=\\
&=\int_0^\infty t^{-n-1}\Big[e^{-\frac{t^k}{k}}-\sum_{j=0}^{m}(-1)^j\frac{t^{kj}}{k^j j!}H(1-t)\Big]\,dt+\sum_{j=0,kj\neq n}^{m}\frac{(-1)^j}{k^j j!}\frac{1}{kj-n}. \qquad (2.11)
\end{aligned}
$$

On the other hand, if there is not any number j in the set $\{1,2,\ldots,m\}$ such that $kj=n$, we can write directly that

$$
\begin{aligned}
\mathrm{N}\!-\!\lim_{\epsilon\to 0}\int_\epsilon^\infty t^{-n-1}\,e^{-\frac{t^k}{k}}\,dt &= \int_0^\infty t^{-n-1}\Big[e^{-\frac{t^k}{k}}-\sum_{j=0}^{m}(-1)^j\frac{t^{kj}}{k^j j!}H(1-t)\Big]\,dt+\\
&+\sum_{j=0}^{m}\frac{(-1)^j}{k^j j!}\frac{1}{kj-n}. \qquad (2.12)
\end{aligned}
$$

This completes the proof of the theorem. □

We will give some examples for making easier to understand our equations.

Example 2.3. *Let us calculate $\Gamma_1(-n)$ for $n=0,1,\ldots$. By taking $n=m$ in the equation (2.11), we have*

$$\Gamma_1(-n)=\int_0^\infty t^{-n-1}\Big[e^{-t}-\sum_{j=0}^{n}(-1)^j\frac{t^j}{j!}H(1-t)\Big]dt-\sum_{j=0}^{n-1}\frac{(-1)^j}{j!(n-j)} \qquad (2.13)$$

for $n=1,2,\ldots$. Also by using the equation (2.10) for $k=1$ and $n=0$, we get

$$\Gamma_1(0)=\int_0^\infty t^{-1}\Big[e^{-t}-H(1-t)\Big]dt. \qquad (2.14)$$

We want to note that the equations (2.13) and (2.14) collide with the ones in [6].

Example 2.4. *For $\Gamma_3(-16/3)$ we use the inequality $-(n+1)k\le x<-nk+1$. Then $-3(n+1)\le -16/3<-3n+1$ is valid for $n=2$. Hence by equation (2.3) we write*

$$
\begin{aligned}
\Gamma_3(-16/3) &= \int_0^\infty t^{-19/3}\Big[e^{-\frac{t^3}{3}}-\sum_{j=0}^{2}(-1)^j\frac{t^{3j}}{3^j j!}H(1-t)\Big]\,dt+\\
&+\sum_{j=0}^{2}\frac{(-1)^j}{3^j j!(3j-16/3)}.
\end{aligned}
$$

Example 2.5. *For $\Gamma_{1/2}(-7/4)$ we can use equation (2.8) for $n = 5$. Because, we have $x = -7/4$, $l = 2$ and $k = 1/2$ and for satisfying $-(n+1)k + 1 \leq x < -nk + 1$ we need $n = 5$. Also, $x \neq -[(n-l)+1]k$, then*

$$\Gamma_{1/2}(-7/4) = \int_0^\infty t^{-11/4}\Big[e^{-\frac{t^{1/2}}{1/2}} - \sum_{j=0}^{5}(-1)^j \frac{t^{j/2}}{(1/2)^j j!} H(1-t)\Big]\, dt + \\ + \sum_{j=0}^{5} \frac{(-1)^j}{(1/2)^j j!(j/2 - 7/4)}.$$

Example 2.6. *We can get $\Gamma_{3/7}(-1)$ in two ways. Firstly, let us use equation (2.6). For this, we need to find the number m satisfying the inequality $\frac{1}{m+1} < k < \frac{1}{m}$. Since $k = 3/7$, we have $m = 2$. Also, since $x = -1$ then we get $n = 4$ for the inequality $-(n+1)k + 1 \leq x < -nk + 1$. Also, $-1 = x \neq -(n-m)k = -2.\frac{3}{7}$. Hence,*

$$\Gamma_{3/7}(-1) = \int_0^\infty t^{-2}\Big[e^{-\frac{t^{3/7}}{3/7}} - \sum_{j=0}^{4}(-1)^j \frac{t^{3/7}}{(3/7)^j j!} H(1-t)\Big]\, dt + \\ + \sum_{j=0}^{4} \frac{(-1)^j}{(3/7)^j j!(3j/7 - 1)}. \tag{2.15}$$

For the second way, we can use equation (2.12). Since $k = 3/7$ and $n = 1$ we get $m = 4$ for the inequality $\frac{n+1}{m+1} \leq k < \frac{n+1}{m}$. Also, there is not any number j in $\{1, 2, 3, 4\}$ such that $3j/7 = -1$. Then we get equation (2.15).

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