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COMMON FIXED POINT THEOREMS FOR L-FUZZY MAPPINGS

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Abstract. In this paper, we prove some common fixed point theorem of L-fuzzy mappings for Θ-contraction in the setting of complete metric space. We also provide an example to show the significance of the investigation of this paper.

1. Introduction and Preliminaries

Answering real-world problems becomes apparently simple with the initiation of fuzzy set theory in 1965 by Zadeh [27], as it helps in making the account of obscurity and imprecision clear and more precise. Later in 1967, Goguen [13] extended this idea to L-fuzzy set theory by replacing the interval $[0, 1]$. There are basically two understandings of the meaning of L, one is when L is a complete lattice equipped with a multiplication $*$ operator satisfying certain conditions as shown in the initial paper [13] and the second understanding of the meaning of L is that L is a completely distributive complete lattice with an order-reversing involution . In 2014, Rashid et al. [22] introduced the notion of β_{F_L}-admissible for a pair of L-fuzzy mappings and utilized it to proved a common L-fuzzy fixed point theorem. For more details on this direction, we refer the reader to [1, 10, 23].

Let $(\mathcal{M}, \sigma)$ be a metric space and $CB(\mathcal{M})$ be the family of nonempty, closed and bounded subsets of $\mathcal{M}$. For$A, B \in CB(\mathcal{M})$, define

$$H(A, B) = \max\left\{\sup_{a\in A} \sigma(a, B), \sup_{b\in B} \sigma(b, A)\right\}$$

where

$$\sigma(x, A) = \inf_{y\in A} \sigma(x, y).$$

Definition 1.1. [13] *A partially ordered set* $(L, \precsim_L)$ *is called a*

$i)$ a lattice, if $a \vee b \in L$, $a \wedge b \in L$ for any $a, b \in L$.
$ii)$ a complete lattice, if $\vee A \in L$, $\wedge A \in L$ for any $A \subseteq L$.

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iii) distributive if $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$, $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ for any $a, b, c \in L$.

Definition 1.2. [13] *Let L be a lattice with top element 1_L and bottom element 0_L and let $a, b \in L$. Then b is called a complement of a, if $a \vee b = 1_L$, and $a \wedge b = 0_L$. If $a \in L$ has a complement element, then it is unique. It is denoted by $\acute{a}$.*

Definition 1.3. [13] *A $L-$fuzzy set A on a nonempty set $\mathcal{M}$ is a function $A : \mathcal{M} \to L$, where L is complete distributive lattice with 1_L and 0_L.*

Remark. *The class of $L-$fuzzy sets is larger than the class of fuzzy sets as an $L-$fuzzy set is a fuzzy set if $L = [0, 1]$.*

The α_L-level set of $L-$fuzzy set A, is denoted by A_{α_L}, and is defined as follows

$$A_{\alpha_L} = \{x : \alpha_L \precsim_L A(x)\} \text{ if } \alpha_L \in L \backslash \{0_L\},$$

$$A_{0_L} = \overline{\{x : 0_L \precsim_L A(x)\}}.$$

Here $cl(B)$ and $\Im_L(Y)$ denote the closure of the set B and L-fuzzy set on Y respectively.

We denote and define the characteristic function χ_{L_A} of a L-fuzzy set A as follows:

$$\chi_{L_A} := \begin{cases} 0_L & \text{if } x \notin A \\ 1_L & \text{if } x \in A \end{cases}.$$

Definition 1.4. *Let $\mathcal{M}$ be an arbitrary set, $\mathcal{N}$ be a metric space. A mapping $\mathcal{Q}$ is called $L-$fuzzy mapping if $\mathcal{Q}$ is a mapping from $\mathcal{M}$ into $\Im_L(\mathcal{N})$. A $L-$fuzzy mapping $\mathcal{Q}$ is a $L-$fuzzy subset on $\mathcal{M} \times \mathcal{N}$ with membership function $\mathcal{Q}(x)(y)$. The function $\mathcal{Q}(x)(y)$ is the grade of membership of y in $\mathcal{Q}(x)$.*

Definition 1.5. *Let $(\mathcal{M}, \sigma)$ be a metric space and $\mathcal{P}, \mathcal{Q}$ be $L-$fuzzy mappings from $\mathcal{M}$ into $\Im_L(\mathcal{M})$. A point $z \in \mathcal{M}$ is called a $L-$fuzzy fixed point of $\mathcal{Q}$ if $z \in [\mathcal{Q}z]_{\alpha_L}$, where $\alpha_L \in L \backslash \{0_L\}$. The point $z \in \mathcal{M}$ is called a common $L-$fuzzy fixed point of $\mathcal{P}$ and $\mathcal{Q}$ if $z \in [\mathcal{P}z]_{\alpha_L} \cap [\mathcal{Q}z]_{\alpha_L}$. When $\alpha_L = 1_L$, it is called a common fixed point of $L-$fuzzy mappings.*

Very recently, Jleli and Samet [18] introduced a new type of contraction called Θ-contraction and established some new fixed point theorems for such contraction in the context of generalized metric spaces.

Definition 1.6. *Let $\Theta : (0, \infty) \to (1, \infty)$ be a function satisfying:*

- (Θ_1) Θ is nondecreasing;
- (Θ_2) for each sequence $\{\alpha_n\} \subseteq R^+$, $\lim_{n\to\infty} \Theta(\alpha_n) = 1$ if and only if $\lim_{n\to\infty}(\alpha_n) = 0$;
- (Θ_3) there exists $0 < h < 1$ and $l \in (0, \infty]$ such that $\lim_{\alpha\to 0^+} \frac{\Theta(\alpha)-1}{\alpha^h} = l$.

A mapping $\mathcal{P} : \mathcal{M} \to \mathcal{M}$ is said to be Θ-contraction if there exist the function Θ satisfying (Θ_1)-(Θ_3) and a constant $k \in (0, 1)$ such that for all $x, y \in \mathcal{M}$,

$$\sigma(\mathcal{P}x, \mathcal{P}y) > 0 \Longrightarrow \Theta(\sigma(\mathcal{P}x, \mathcal{P}y)) \leq [\Theta(\sigma(x, y))]^k. \tag{1.1}$$

Theorem 1.7. [18] *Let $(\mathcal{M}, \sigma)$ be a complete metric space and $\mathcal{P} : \mathcal{M} \to \mathcal{M}$ be a Θ-contraction, then $\mathcal{P}$ has a unique fixed point.*

They showed that any Banach contraction is a particular case of Θ-contraction while there are Θ-contractions which are not Banach contractions.To be consistent with Samet et al. [18], we denote by the Ψ set of all functions $\Theta : (0,\infty) \to (1,\infty)$ satisfying the above conditions (Θ_1)-(Θ_3).

Later on Altune et al.[14] modified the above definitions by adding a general condition (Θ_4) which is given in this way:

(F_4) $\Theta(\inf A) = \inf \Theta(A)$ for all $A \subset (0,\infty)$ with $\inf A > 0$.

Following Altune et al.[14], we represent the set of all continuous functions $\Theta : \mathbb{R}^+ \to \mathbb{R}$ satisfying $(\Theta_1) - (\Theta_4)$ conditions by Ω.

For more details on Θ-contraction, we refer the reader to [6, 21, 26].

In this paper, we use a generalized Θ-contraction to obtain common fixed points for L- fuzzy mappings in the setting of metric spaces.

For the sake of convenience, we first state some known results for subsequent use in the next section

Lemma 1.8. *Let $(\mathcal{M},\sigma)$ be a metric space and $A,B \in CB(\mathcal{M})$, then for each $a \in A$,*

$$\sigma(a,B) \leq H(A,B).$$

2. Main Results

In this way, we state and prove a common fixed point theorem for L-fuzzy mappings.

Theorem 2.1. *Let $(\mathcal{M},\sigma)$ be a complete metric space and let $\mathcal{P},\mathcal{Q}$ be L-fuzzy mappings from $\mathcal{M}$ into $\Im_L(\mathcal{M})$ and for each $\alpha_L \in L\backslash\{0_L\}$, $[\mathcal{P}x]_{\alpha_L(x)}$, $[\mathcal{Q}y]_{\alpha_L(y)}$ are nonempty closed bounded subsets of $\mathcal{M}$. If there exist some $\Theta \in \Omega$ and $k \in (0,1)$ such that*

$$\Theta\left(H\left([\mathcal{P}x]_{\alpha_L(x)}, [\mathcal{Q}y]_{\alpha_L(y)}\right)\right) \leq \Theta(\sigma(x,y))^k \tag{2.1}$$

for all $x,y \in \mathcal{M}$ with $H\left([\mathcal{P}x]_{\alpha_L(x)}, [\mathcal{Q}y]_{\alpha_L(y)}\right) > 0$. Then $\mathcal{P}$ and $\mathcal{Q}$ have a common L-fuzzy fixed point.

Proof. Let x_0 be an arbitrary point in $\mathcal{M}$, then by hypotheses there exists $\alpha_L(x_0) \in L\backslash\{0_L\}$ such that $[\mathcal{P}x_0]_{\alpha_L(x_0)}$ is a nonempty closed bounded subset of $\mathcal{M}$ and let $x_1 \in [\mathcal{P}x_0]_{\alpha_L(x_0)}$. For this x_1 there exists $\alpha_L(x_1) \in L\backslash\{0_L\}$ such that $[\mathcal{Q}x_1]_{\alpha_L(x_1)}$ is a nonempty, closed and bounded subset of $\mathcal{M}$. By Lemma 1.8, (Θ_1) and (2.1), we have

$$\Theta(d\left(x_1, [Tx_1]_{\alpha_L(x_1)}\right) \leq \Theta\left(H\left([Sx_0]_{\alpha_L(x_0)}, [Tx_1]_{\alpha_L(x_1)}\right)\right)$$

which implies

$$\Theta(d\left(x_1, [Tx_1]_{\alpha_L(x_1)}\right) \leq \Theta(d(x_0,x_1))^k. \tag{2.2}$$

From (Θ_4), we know that

$$\Theta\left(\sigma\left(x_1, [\mathcal{Q}x_1]_{\alpha_{\mathcal{Q}}(x_1)}\right)\right) = \inf_{y \in [\mathcal{Q}x_1]_{\alpha_L(x_1)}} \Theta(\sigma(x_1,y)).$$

Thus from (2.2), we get

$$\inf_{y \in [\mathcal{Q}x_1]_{\alpha_L(x_1)}} \Theta(\sigma(x_1,y)) \leq [\Theta(\sigma(x_0,x_1))]^k. \tag{2.3}$$

Then, from (2.3), there exists $x_2 \in [\mathcal{Q}x_1]_{\alpha_L(x_1)}$ such that

$$\Theta(\sigma(x_1, x_2)) \leq [\Theta(\sigma(x_0, x_1)]^k. \tag{2.4}$$

For this x_2 there exists $\alpha_L(x_2) \in L\backslash\{0_L\}$ such that $[\mathcal{P}x_2]_{\alpha_L(x_2)}$ is a nonempty closed bounded subset of $\mathcal{M}$. By Lemma 1.8, (Θ_1) and (2.1), we have

$$\begin{aligned}\Theta\left(\sigma\left(x_2, [\mathcal{P}x_2]_{\alpha_L(x_2)}\right)\right) &\leq \Theta(H\left([\mathcal{Q}x_1]_{\alpha_L(x_1)}, [\mathcal{P}x_2]_{\alpha_L(x_2)}\right) = \Theta(H\left([\mathcal{P}x_2]_{\alpha_L(x_2)}, [\mathcal{Q}x_1]_{\alpha_L(x_1)}\right) \\ &\leq \Theta(\sigma(x_2, x_1))^k = \Theta(\sigma(x_1, x_2))^k\end{aligned}$$

From (Θ_4), we know that

$$\Theta\left[\sigma\left(x_2, [\mathcal{P}x_2]_{\alpha_L(x_2)}\right)\right] = \inf_{y_1 \in [\mathcal{P}x_2]_{\alpha_L(x_2)}} \Theta(\sigma(x_2, y_1))$$

Thus

$$\inf_{y_1 \in [\mathcal{P}x_2]_{\alpha_L(x_2)}} \Theta(\sigma(x_2, y_1)) \leq \Theta\left[\sigma(x_1, x_2)\right]^k. \tag{2.5}$$

Then, from (2.5), there exists $x_3 \in [\mathcal{P}x_2]_{\alpha_L(x_2)}$ such that

$$\Theta(\sigma(x_2, x_3)) \leq [\Theta(\sigma(x_1, x_2)]^k. \tag{2.6}$$

So, continuing recursively, we obtain a sequence $\{x_n\}$ in $\mathcal{M}$ such that $x_{2n+1} \in [\mathcal{P}x_{2n}]_{\alpha_L(x_{2n})}$ and $x_{2n+2} \in [\mathcal{Q}x_{2n+1}]_{\alpha_L(x_{2n+1})}$ and

$$\Theta(\sigma(x_{2n+1}, x_{2n+2})) \leq [\Theta(\sigma(x_{2n}, x_{2n+1})]^k \tag{2.7}$$

and

$$\Theta(\sigma(x_{2n+2}, x_{2n+3})) \leq [\Theta(\sigma(x_{2n+1}, x_{2n+2})]^k \tag{2.8}$$

for all $n \in \mathbb{N}$. From (2.7) and (2.8), we have

$$\Theta(\sigma(x_n, x_{n+1})) \leq [\Theta(\sigma(x_{n-1}, x_n)]^k \tag{2.9}$$

which further implies that

$$\Theta(\sigma(x_n, x_{n+1})) \leq [\Theta(\sigma(x_{n-1}, x_n)]^k \leq [\Theta(\sigma(x_{n-2}, x_{n-1})]^{k^2} \leq ... \leq [\Theta(\sigma(x_0, x_1)]^{k^n} \tag{2.10}$$

for all $n \in \mathbb{N}$. Since $\Theta \in \Omega$, so by taking limit as $n \to \infty$ in (2.10) we have,

$$\lim_{n\to\infty} \Theta(\sigma(x_n, x_{n+1})) = 1 \tag{2.11}$$

which implies that

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = 0 \tag{2.12}$$

by (Θ_2). From the condition (Θ_3), there exist $0 < r < 1$ and $l \in (0, \infty]$ such that

$$\lim_{n\to\infty} \frac{\Theta(\sigma(x_n, x_{n+1})) - 1}{\sigma(x_n, x_{n+1})^r} = l. \tag{2.13}$$

Suppose that $l < \infty$. In this case, let $\beta = \frac{l}{2} > 0$. From the definition of the limit, there exists $n_0 \in \mathbb{N}$ such that

$$|\frac{\Theta(\sigma(x_n, x_{n+1})) - 1}{\sigma(x_n, x_{n+1})^r} - l| \leq \beta$$

for all $n > n_0$. This implies that

$$\frac{\Theta(\sigma(x_n, x_{n+1})) - 1}{\sigma(x_n, x_{n+1})^r} \geq l - \beta = \frac{l}{2} = \beta$$

for all $n > n_0$. Then

$$n\sigma(x_n, x_{n+1})^r \leq An[\Theta(\sigma(x_n, x_{n+1})) - 1] \tag{2.14}$$

for all $n > n_0$, where $\alpha = \frac{1}{\beta}$. Now we suppose that $l = \infty$. Let $\beta > 0$ be an arbitrary positive number. From the definition of the limit, there exists $n_0 \in \mathbb{N}$ such that

$$\beta \leq \frac{\Theta(\sigma(x_n, x_{n+1})) - 1}{\sigma(x_n, x_{n+1})^r}$$

for all $n > n_0$. This implies that

$$n\sigma(x_n, x_{n+1})^r \leq \alpha n[\Theta(\sigma(x_n, x_{n+1})) - 1]$$

for all $n > n_0$, where $\alpha = \frac{1}{\beta}$. Thus, in all cases, there exist $\alpha > 0$ and $n_0 \in \mathbb{N}$ such that

$$n\sigma(x_n, x_{n+1})^r \leq \alpha n[\Theta(\sigma(x_n, x_{n+1})) - 1] \tag{2.15}$$

for all $n > n_0$. Thus by (2.10) and (2.15), we get

$$n\sigma(x_n, x_{n+1})^r \leq \alpha n([(\Theta\sigma(x_0, x_1))]^{r^n} - 1). \tag{2.16}$$

Letting $n \to \infty$ in the above inequality, we obtain

$$\lim_{n\to\infty} n\sigma(x_n, x_{n+1})^r = 0.$$

Thus, there exists $n_1 \in \mathbb{N}$ such that

$$\sigma(x_n, x_{n+1}) \leq \frac{1}{n^{1/r}} \tag{2.17}$$

for all $n > n_1$. Now we prove that $\{x_n\}$ is a Cauchy sequence. For $m > n > n_1$ we have,

$$\sigma(x_n, x_m) \leq \sum_{i=n}^{m-1} \sigma(x_i, x_{i+1}) \leq \sum_{i=n}^{m-1} \frac{1}{i^{1/r}} \leq \sum_{i=1}^{\infty} \frac{1}{i^{1/r}}. \tag{2.18}$$

Since, $0 < r < 1$, then $\sum_{i=1}^{\infty} \frac{1}{i^{1/r}}$ converges. Therefore, $\sigma(x_n, x_m) \to 0$ as $m, n \to \infty$. Thus we proved that $\{x_n\}$ is a Cauchy sequence in $(\mathcal{M}, \sigma)$. The completeness of $(\mathcal{M}, \sigma)$ ensures that there exists $u \in X$ such that, $\lim_{n\to\infty} x_n \to u$. Now, we prove that $u \in [\mathcal{Q}u]_{\alpha_L(u)}$. We suppose on the contrary that $u \notin [\mathcal{Q}u]_{\alpha_L(u)}$, then there exist a $n_0 \in \mathbb{N}$ and a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\sigma(x_{2n_k+1}, [\mathcal{Q}u]_{\alpha_L(u)}) > 0$ for all $n_k \geq n_0$. Since $\sigma(x_{2n_k+1}, [\mathcal{Q}u]_{\alpha_L(u)}) > 0$ for all $n_k \geq n_0$, so by (Θ_1), we have

$$\begin{aligned} \Theta\left[\sigma(x_{2n_k+1}, [\mathcal{Q}u]_{\alpha_L(u)})\right] &\leq \Theta\left[H([\mathcal{P}x_{2n_k}]_{\alpha_L(x_{2n_k})}, [\mathcal{Q}u]_{\alpha_L(u)})\right] \\ &\leq [\Theta(\sigma(x_{2n_k}, u))]^k. \end{aligned}$$

Letting $n \to \infty$, in above inequality and using the continuity of Θ, we have

$$\Theta\left[\sigma(u, [\mathcal{Q}u]_{\alpha_L(u)})\right] \leq 0$$

Hence $u \in [\mathcal{Q}u]_{\alpha_L(u)}$. Similarly, one can easily prove that $u \in [\mathcal{P}u]_{\alpha_L(u)}$. Thus $u \in [\mathcal{P}u]_{\alpha_L(u)} \cap [\mathcal{Q}u]_{\alpha_L(u)}$. □

The following result is a direct consequence of above theorem.

Theorem 2.2. *Let $(\mathcal{M},\sigma)$ be a complete metric space and let $\mathcal{P}$ be L-fuzzy mapping from $\mathcal{M}$ into $\Im_L(\mathcal{M})$ and for each $\alpha_L \in L\backslash\{0_L\}$, $[\mathcal{P}x]_{\alpha_L(x)}$, $[\mathcal{P}y]_{\alpha_L(y)}$ are nonempty closed bounded subsets of $\mathcal{M}$. If there exist some $\Theta \in \Omega$ and $k \in (0,1)$ such that*

$$\Theta\left(H\left([\mathcal{P}x]_{\alpha_L(x)}, [\mathcal{P}y]_{\alpha_L(y)}\right)\right) \leq \Theta(\sigma(x,y))^k$$

for all $x, y \in \mathcal{M}$ with $H\left([\mathcal{P}x]_{\alpha_L(x)}, [\mathcal{P}y]_{\alpha_L(y)}\right) > 0$. Then $\mathcal{P}$ has an L-fuzzy fixed point.

Corollary 2.3. *Let $(\mathcal{M},\sigma)$ be a complete metric space and let $\mathcal{P},\mathcal{Q}$ be fuzzy mappings from $\mathcal{M}$ into $\Im(\mathcal{M})$ and for each $\alpha(x) \in (0,1]$, $[\mathcal{P}x]_{\alpha(x)}$, $[\mathcal{Q}y]_{\alpha(y)}$ are nonempty closed bounded subsets of $\mathcal{M}$. If there exist some $\Theta \in \Omega$ and $k \in (0,1)$ such that*

$$\Theta\left(H\left([\mathcal{P}x]_{\alpha(x)}, [\mathcal{Q}y]_{\alpha(y)}\right)\right) \leq \Theta(\sigma(x,y))^k$$

for all $x, y \in \mathcal{M}$ with $H\left([\mathcal{P}x]_{\alpha(x)}, [\mathcal{Q}y]_{\alpha(y)}\right) > 0$. Then $\mathcal{P}$ and $\mathcal{Q}$ have a common fuzzy fixed point.

Proof. Consider an L-fuzzy mapping $\mathcal{J} : \mathcal{M} \to \Im_L(\mathcal{M})$ defined by

$$\mathcal{J}x = \chi_{L_{\mathcal{P}(x)}}.$$

Then for $\alpha_L \in L\backslash\{0_L\}$, we have

$$[\mathcal{J}x]_{\alpha_L(x)} = \mathcal{P}x.$$

Hence by Theorem 2.1 we follow the result. □

Example 2.4. *Let $\mathcal{M} = [0,1]$, $\sigma(x,y) = |x-y|$, whenever $x, y \in \mathcal{M}$. Then $(\mathcal{M},\sigma)$ is a complete metric space. Let $L = \{\eta,\omega,\tau,\kappa\}$ with $\eta \preceq_L \omega \preceq_L \kappa$ and $\eta \preceq_L \tau \preceq_L \kappa$, where ω and τ are not comparable, then $(L,\preceq_L)$ is a complete distributive lattice. Define a pair of mappings $\mathcal{P},\mathcal{Q} : \mathcal{M} \to \Im_L(\mathcal{M})$ as follows:*

$$\mathcal{P}(x)(t) = \begin{cases} \kappa \text{ if } 0 \leq t \leq \frac{x}{6} \\ \omega \text{ if } \frac{x}{6} < t \leq \frac{x}{3} \\ \tau \text{ if } \frac{x}{3} < t \leq \frac{x}{2} \\ \eta \text{ if } \frac{x}{2} < t \leq 1 \end{cases},$$

$$\mathcal{Q}(x)(t) = \begin{cases} \kappa \text{ if } 0 \leq t \leq \frac{x}{12} \\ \eta \text{ if } \frac{x}{12} < t \leq \frac{x}{8} \\ \omega \text{ if } \frac{x}{8} < t \leq \frac{x}{4} \\ \tau \text{ if } \frac{x}{4} < t \leq 1 \end{cases}.$$

Let $\Theta(t) = e^{\sqrt{t}} \in \Omega$ for $t > 0$. And for all $x \in \mathcal{M}$, there exists $\alpha_L(x) = \kappa$, such that

$$[\mathcal{P}x]_{\alpha_L(x)} = \left[0, \frac{x}{6}\right], \qquad [\mathcal{Q}x]_{\alpha_L(x)} = \left[0, \frac{x}{12}\right].$$

and all conditions of Theorem 2.1 are satisfied and 0 is a common fixed point of $\mathcal{P}$ and $\mathcal{Q}$.

3. Conclusion

In this paper, we obtained the common fuzzy fixed points of L-fuzzy mappings satisfying Θ-contraction in the context of metric space. In the process, we generalize several well known recent and classical results.

Conflict of Interests

The authors declare that they have no competing interests.

Authors' Contribution

All authors contributed equally and significantly in writing this paper. All authors read and approved the final paper.

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References

[1] M. S. Abdullahi and A. Azam, *L-fuzzy Fixed Point Theorems for L-fuzzy Mappings via β_{F_L}-admissible with Applications*, Journal of Uncertainty Analysis and Applications 2017 **5**:2

[2] J. Ahmad, A. E. Al-Mazrooei, Y. J. Cho, Y. O. Yang, *Fixed point results for generalized Θ-contractions*, J. Nonlinear Sci. Appl., **10** (2017), 2350–2358

[3] J. Ahmad, A. Al-Rawashdeh and A. Azam, *New Fixed Point Theorems for Generalized F-Contractions in Complete Metric Spaces*, Fixed Point Theory and Applications 2015, 2015:**80**

[4] Z. Aslam, J. Ahmad, N. Sultana, *New common fixed point theorems for cyclic compatible contractions*, J. Math. Anal., **8** (2017), 1–12.

[5] J. Ahmad, A. E. Al-Mazrooei, I. Altun, *Generalized Θ-contractive fuzzy mappings*, Journal of Intelligent and Fuzzy Systems, Vol **35** (2)(2018), 1935-1942

[6] A. Ahmad, A. Al-Rawashdeh, A. Azam, *Fixed point results for $\{\alpha, \xi\}$-expansive locally contractive mappings*, Journal of Inequalities and Applications 2014, 2014:**364**

[7] L. A. Alnaser, D. Lateef, H. A. Fouad, J. Ahmad, *Relation theoretic contraction results in F-metric spaces*, J. Nonlinear Sci. Appl., **12** (2019), 337–344

[8] A. Al-Rawashdeh and J. Ahmad, *Common Fixed Point Theorems for JS- Contractions*, Bull. Math. Anal. Lapp.,Volume **8** Issue 4(2016), Pages 12-22.

[9] H. H. Al-Sulami, J. Ahmad, N. Hussain, A. Latif, *Relation Theoretic (Θ,R) Contraction Results with Applications to Nonlinear Matrix Equations*, Symmetry 2018, 10, **767**; doi:10.3390/sym10120767

[10] A. Azam, N. Mahmood, M. Rashid, M. Pavlović, *L-fuzzy fixed points in cone metric spaces*, J. Adv. Math. Stud, Vol. **9**(2016), No. 1, 121-131

[11] S. Banach, *Sur les operations dans les ensembles abstraits et leur applications aux equations integrales*, Fundam. Math. **3**(1922), 133-181.

[12] G. Durmaz, *Some theorems for new type multivalued contractive maps on metric space*, Turkish Journal of Mathematics, DOI: 10.3906/mat-1510-**75**

[13] J. A. Goguen, *L-fuzzy sets*, Journal of Mathematical Analysis and Applications, vol. **18**, pp. 145–174, 1967.

[14] HA. Hancer, G. Minak, I. Altun, *On a broad category of multivalued weakly Picard operators.* Fixed Point Theory, **18**(2017), No. 1, 229-236

[15] S. Heilpern, *Fuzzy mappings and fixed point theorem*, J. Math. Anal. Appl. **83** (1981), no. 2, 566-569.

[16] N. Hussain, A. E. Al-Mazrooei, J. Ahmad, *Fixed point results for generalized (α-η)-Θ contractions with applications*, J. Nonlinear Sci. Appl., **10** (2017), 4197–4208

[17] N. Hussain, J. Ahmad and A. Azam, *On Suzuki-Wardowski type fixed point theorems*, J. Nonlinear Sci. Appl. **8** (2015), 1095-1111

[18] M. Jleli, B. Samet, *A new generalization of the Banach contraction principle.* J. Inequal. Appl. 2014, **38** (2014)

[19] M.A. Kutbi, J. Ahmad and A. Azam, *On fixed points of $\alpha - \psi$- contractive multi- valued mappings in cone metric spaces*, Abstract and Applied Analysis, Volume 2013, Article ID 313782.
[20] Jr. Nadler, *Multi-valued contraction mappings. $Pacific\ J.Math.$* **30**(1969), 475-478.
[21] W. Onsod , T. Saleewong, J. Ahmad, A. E. Al-Mazrooei and P. Kumam, *Fixed points of a Θ-contraction on metric spaces with a graph*, Commun. Nonlinear Anal. **2** (2016), 139–14
[22] M. Rashid, A. Azam and N. Mehmood, *L-fuzzy fixed points theorems for L-fuzzy mappings via β_{F_L}-admissible pair*. Sci. World J. **2014**(2014), 1–8
[23] M. Rashid, M.A. Kutbi and A. Azam, *Coincidence theorems via alpha cuts of L-fuzzy sets with applications*, Fixed Point Theory Appl. **212**(2014), 1–16 .
[24] M. Riaz, and M. R. Hashmi, *Fixed points of fuzzy neutrosophic soft mapping with decision-making*, Fixed Point Theory and Applications 2018, no. 1 (2018): **7**.
[25] A.F. Sayed , J. Ahmad , *Some fixed point theorems for fuzzy soft contractive mappings in fuzzy soft metric spaces*, Italian Journal of Pure and Applied Mathematics, **40**(2018), 200-214
[26] F. Vetro, *A generalization of Nadler fixed point theorem*, Carpathian J. Math, **31** (2015), 403 - 410
[27] LA. Zadeh, *Fuzzy sets*, Inf. Control. **8**(3)(1965), 338–353

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