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COMPLEX VALUED M-METRIC SPACES AND RELATED FIXED POINT RESULTS VIA COMPLEX C-CLASS FUNCTIONS

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Abstract. In this paper, we initiate the concept of complex valued M-metric spaces which extends the notion of M-metric spaces introduced by Asadi *et al.* [3]. Then by using complex C-class functions, some fixed point theorems are established in such spaces. The obtained results generalize and improve some existing ones in the literature

1. Introduction

The fixed point theorem, generally known as the Banach Contraction Principle, appeared in explicit form in Banach thesis in 1922. Fixed point theory is also very famous due to its variety of applications in numerous areas such as engineering, computer science, economics, etc. The contractive type conditions play an important role in the fixed point theory.

Many researchers have extended and generalized the Banach Contraction Principle because it is the heart of this theory. In 1994, in [8] Matthews introduced the notion of a partial metric space and proved the contraction principle of Banach in this new framework. Next, many fixed point theorems in partial metric spaces have been given by several mathematicians. In 2012, Haghi *et al.* published [6] a paper which stated that we should be careful on some fixed point results on partial metric spaces. They showed that many fixed point generalizations on partial metric spaces can be obtained from the corresponding results in metric spaces. For other results in this setting, see [1, 2, 13, 14, 17, 18, 20, 21].

In 2014, Asadi *et al.* [3] extended the p-metric space to a M-metric space, and proved some theorems for generalized contractions. For more informations and results on M-metric spaces, see also [4, 5, 9, 10, 11, 12].

In 2011, Azam *et al.* [19] defined the new notion of complex valued metric spaces and obtained some common fixed point theorems. Rao *et al.* [22] also presented the complex valued b-metric spaces.

This paper is organized as follows: In section 2, we give the required information, notions and definitions about M-metric spaces and complex C-class functions. In

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section 3, we introduce the concept of complex valued M-metric spaces as a generalization of M-metric spaces. Next some properties and examples of such spaces are given. In section 4, the topology for complex valued M-metric spaces is discussed and several essential lemmas are proved. Finally, in section 5, our main result is brought and by applying complex C-class functions, some fixed point results are established in complex valued M-metric spaces.

2. Preliminaries

To begin with, we give some basic definitions, notations and theorems which will be used later.

Let X be a non empty set and $m : X \times X \to \mathbb{R}^+$ be a given function. For $x, y \in X$, consider

(1) $m_{xy} := \min\{m(x,x), m(y,y)\}$,
(2) $M_{xy} := \max\{m(x,x), m(y,y)\}$.

Definition 2.1. ([3]) *Let X be a non empty set. A function $m : X \times X \to \mathbb{R}^+$ is called a M-metric if the following conditions are satisfied:*

(m1) $m(x,x) = m(y,y) = m(x,y) \iff x = y$;
(m2) $m_{xy} \leq m(x,y)$;
(m3) $m(x,y) = m(y,x)$;
(m4) $(m(x,y) - m_{xy}) \leq (m(x,z) - m_{xz}) + (m(z,y) - m_{zy})$.

Then the pair (X, m) is called a M-metric space.

Azam *et al.* [19] introduced the notion of complex valued metric spaces. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2) \text{ and } Im(z_1) \leq Im(z_2).$$

If one of the following conditions is satisfied:

(C_1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$;
(C_2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$;
(C_3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$;
(C_4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$;

it follows that $z_1 \precsim z_2$. Note that we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of $(C_2), (C_3)$ and (C_4) is satisfied and we write $z_1 \prec z_2$ if only (C_4) is satisfied. The following statements hold:

(1) If $a, b \in \mathbb{R}$ with $0 \leq a \leq b$, then $az \precsim bz$ for all $z \in \mathbb{C}$.
(2) If $0 \precsim z_1 \precnsim z_2$, then $|z_1| < |z_2|$.
(3) If $z_1 \precsim z_2$ and $z_2 \prec z_3$, then $z_1 \prec z_3$.

Definition 2.2. ([19]) *Let X be a non empty set. A function $d : X \times X \to \mathbb{C}$ is called a complex valued metric on X if the following conditions are satisfied:*

(c1) $0 \precsim d(x,y)$ *for all* $x, y \in X$ *and* $d(x,y) = 0 \Leftrightarrow x = y$;
(c2) $d(x,y) = d(y,x)$ *for all* $x, y \in X$;
(c3) $d(x,y) \precsim d(x,z) + d(z,y)$.

Then the pair (X, d) is called a complex valued metric space.

Definition 2.3. ([19]) *Suppose that (X, d) is a complex valued metric space.*

(1) *We say that the sequence $\{x_n\}$ is Cauchy if for every $0 \prec c \in \mathbb{C}$ there exists an integer N such that $d(x_n, x_m) \prec c$ for all $n, m \geq N$.*
(2) *We say that x_n converges to an element $x \in X$ if for every $0 \prec c \in \mathbb{C}$, there exists an integer N such that $d(x_n, x) \prec c$ for all $n \geq N$.*
(3) *We say that (X, d) is complete if every Cauchy sequence in X converges to a point in X.*

Very recently, Ansari *et al.* [16] introduced the concept of complex C-class functions as follows:

Definition 2.4. ([16]) *Let $S = \{z \in \mathbb{C} : 0 \precsim z\}$. A continuous function $F : S^2 \to \mathbb{C}$ is called a complex C-class function if for any $s, t \in S$, the following conditions hold:*
(1) $F(s, t) \precsim s$;
(2) $F(s, t) = s$ implies that either $s = 0$ or $t = 0$.

An extra condition on F is that $F(0, 0) = 0$ could be imposed in some cases if required. The same letter $\mathcal{C}$ will denote the class of all complex C-*class* functions.
Now, we give examples of some complex C-*class* functions.

Example 2.5. *Let $S = \{z \in \mathbb{C} : 0 \precsim z\}$. The following are examples of complex C-class functions:*

(1) *$F(s, t) = s - t$ where $s, t \in S$.*
(2) *$F(s, t) = ms$, for some $m \in (0, 1)$ and $s, t \in S$.*
(3) *$F(s, t) = s - \phi(s)$, where $\phi : S \to S$ is continuous, $\phi(0) = 0$ and $\phi(t) \succ 0$ if $t \succ 0$.*
(4) *$F(s, t) = s\beta(s)$, where $\beta : S \to [0, 1)$ is continuous and $s, t \in S$.*

3. Complex valued M-metric spaces

First, note that $z_1, z_2 \in \mathbb{C}$ are called comparable if and only if

$$z_1 \precsim z_2 \quad \text{or} \quad z_2 \precsim z_1.$$

If all elements of a set $D \subseteq \mathbb{C}$ are comparable pairwise, then we can define the "min" and "max" for D. Now, we initiate the concept of complex valued M-metric spaces and give some related properties.

Definition 3.1. *Let X be a non empty set. A function $m : X \times X \to \mathbb{C}$ is called a complex valued M-metric if the following conditions are satisfied:*

(cm1) *$0 \precsim m(x, y)$ for all $x, y \in X$ and $m(x, x) = m(y, y) = m(x, y) \iff x = y$;*
(cm2) *$m(x, x)$ and $m(y, y)$ are comparable for all $x, y \in X$;*
(cm3) *$m_{xy} \precsim m(x, y)$ for all $x, y \in X$, where $m_{xy} = \min\{m(x, x), m(y, y)\}$;*
(cm4) *$m(x, y) = m(y, x)$ for all $x, y \in X$;*
(cm5) *$(m(x, y) - m_{xy}) \precsim (m(x, z) - m_{xz}) + (m(z, y) - m_{zy})$ for all $x, y, z \in X$.*

Then the pair (X, m) is called a complex valued M-metric space.

Remark 3.2. *Let (X, m) be a complex valued M-metric space. For every $x, y, z \in X$, we have*

(1) *$0 \precsim M_{xy} + m_{xy} = m(x, x) + m(y, y)$;*
(2) *$0 \precsim M_{xy} - m_{xy} = (m(x, x) - m(y, y)) \vee (m(y, y) - m(x, x))$;*

(3) $M_{xy} - m_{xy} \precsim (M_{xz} - m_{xz}) + (M_{zy} - m_{zy})$.

We state the following examples.

Example 3.3. *Let $X = [0, \infty)$. Then $m(x, y) = \frac{x+y}{2} + i\frac{x+y}{2}$ on X is a complex valued M-metric.*

Example 3.4. *Let $X = \{1, 2, 3\}$. Define m the complex valued M-metric on X as*

$$m(1, 2) = m(2, 1) = m(1, 1) = 8 + i8,$$
$$m(1, 3) = m(3, 1) = m(3, 2) = m(2, 3) = 7 + i7,$$
$$m(2, 2) = 9 + i9 \text{ and } m(3, 3) = 5 + i5.$$

If $D(x, y) = m(x, y) - m_{xy}$, then $m(1, 2) = m_{12} = 8 + i8$. We have $D(1, 2) = 0$ and $1 \neq 2$, which means that D is not a complex valued metric.

Example 3.5. *Let (X, d) be a complex valued metric space. Take $\phi : \{z \in \mathbb{C} : 0 \precsim z\} \to \{z \in \mathbb{C} : \phi(0) \precsim z\}$ a one to one and nondecreasing (or strictly increasing) mapping such that $\phi(0) \succsim 0$ and*

$$\phi(x + y) \precsim \phi(x) + \phi(y) - \phi(0), \quad \forall x, y \succsim 0.$$

Then $m(x, y) = \phi(d(x, y))$ is a complex valued M-metric.

Proof. (cm1), (cm2), (cm3) and (cm4) are clear. For (cm5), we have

$$\begin{aligned} \phi(d(x, y)) &\precsim \phi(d(x, z) + d(z, y)) \\ &\precsim \phi(d(x, z)) + \phi(d(z, y)) - \phi(0). \end{aligned}$$

Thus

$$\begin{aligned} &(\phi(d(x, y)) - \phi(0)) \precsim (\phi(d(x, z)) - \phi(0)) + (\phi(d(z, y)) - \phi(0)) \\ &\Rightarrow (m(x, y) - m_{xy}) \precsim (m(x, z) - m_{xz}) + (m(z, y) - m_{zy}). \end{aligned}$$

□

Example 3.6. *Let (X, d) be a complex valued metric space. For $a, b > 0$, $m(x, y) = ad(x, y) + ib$ is a complex valued M-metric. It suffices to take $\phi(t) = at + ib$, for all $t \in \{z \in \mathbb{C} : 0 \precsim z\}$, in Example 3.5.*

The next example states that m^s and m^w are complex valued metrics.

Example 3.7. *Let m be a complex valued M-metric. Consider*

(1) $m^w(x, y) = m(x, y) - 2m_{xy} + M_{xy}$;
(2) $m^s(x, y) = m(x, y) - m_{xy}$ *if* $x \neq y$ *and* $m^s(x, y) = 0$ *if* $x = y$.

Then m^w and m^s are complex valued metrics.

Proof. We have $m^w(x, y) \succsim 0$ and if $m^w(x, y) = 0$, then

$$m(x, y) = 2m_{xy} - M_{xy}. \tag{1}$$

But, from equation (1) and $m_{xy} \precsim m(x, y)$, we get $m_{xy} = M_{xy} = m(x, x) = m(y, y)$. Again, by equation (1), we obtain $m(x, y) = m(x, x) = m(y, y)$, and therefore $x = y$. For the triangle inequality, it suffices to consider Remark 3.2 and $(cm6)$. Similarly, we can show that m^s is a complex valued metric. □

Remark 3.8. *Let m be a complex valued M-metric. For every $x, y \in X$, we have*

(1) $m(x, y) - M_{xy} \precsim m^w(x, y) \precsim m(x, y) + M_{xy}$;
(2) $(m(x, y) - M_{xy}) \precsim m^s(x, y) \precsim m(x, y)$.

4. Topology for complex valued M-metric spaces

It is clear that each complex valued M-metric m on a non-empty set X generates a T_0 topology τ_m on X. Let $0 \prec \varepsilon \in \mathbb{C}$. The set

$$\{B_m(x,\varepsilon) : x \in X, \varepsilon \succ 0\},$$

where

$$B_m(x,\varepsilon) = \{y \in X : m(x,y) \prec m_{xy} + \varepsilon\},$$

for all $x \in X$ and $\varepsilon \succ 0$, forms the basis of τ_m.

Definition 4.1. *Let (X,m) be a complex valued M-metric space, $x \in X$ and $\{x_n\}$ be a sequence in X. Then*

(1) *$\{x_n\}$ m-converges to x whenever for every $0 \prec c \in \mathbb{C}$, there is a natural number N such that $m(x_n,x) - m_{x_nx} \prec c$ for all $n \geq N$. We denote this by*
$$\lim_{n\to\infty}(m(x_n,x) - m_{x_nx}) = 0. \tag{2}$$
(2) *$\{x_n\}$ is a m-Cauchy sequence whenever for every $0 \prec c \in \mathbb{C}$, there is a natural number N such that $m(x_n,x_m) - 2m_{x_nx_m} + M_{x_nx_m} \prec c$, for all $m,n \geq N$.*
(3) *(X,m) is complete if every m-Cauchy sequence $\{x_n\}$ in X m-converges, with respect to τ_m, to a point $x \in X$ such that*
$$\lim_{n\to\infty}(m(x_n,x) - 2m_{x_nx} + M_{x_nx}) = 0.$$

Lemma 4.2. *Let (X,m) be a complex valued M-metric space. Then*

(1) *$\{x_n\}$ is a m-Cauchy sequence in (X,m) if and only if it is a Cauchy sequence in the complex valued metric space (X,m^w);*
(2) *A complex valued M-metric space (X,m) is complete if and only if the complex valued metric space (X,m^w) is complete. Furthermore,*

$$\lim_{n\to\infty} m^w(x_n,x) = 0$$
$$\iff \left(\lim_{n\to\infty}(m(x_n,x) - m_{x_nx}) = 0 \quad \& \quad \lim_{n\to\infty}(M_{x_nx} - m_{x_nx}) = 0\right).$$

Proof. By Definitions 2.3, 4.1 and definition of m^w, the statements (1) and (2) are easy to obtain. □

Likewise, above definition also holds for m^s.

Lemma 4.3. *Assume that $x_n \to x$ and $y_n \to y$ as $n \to \infty$ in a complex valued M-metric space (X,m). Then*

$$\lim_{n\to\infty}(m(x_n,y_n) - m_{x_ny_n}) = m(x,y) - m_{xy}.$$

Proof. It suffices to write that

$$(m(x_n,y_n) - m_{x_ny_n}) - (m(x,y) - m_{xy}) \precsim (m(x_n,x) - m_{x_nx}) + (m(y,y_n) - m_{yy_n});$$

and

$$(m(x,y) - m_{xy}) - (m(x_n,y_n) - m_{x_ny_n}) \precsim (m(x_n,x) - m_{x_nx}) + (m(y,y_n) - m_{yy_n}).$$

□

From Lemma 4.3, we can deduce the following.

Lemma 4.4. *Assume that $x_n \to x$ as $n \to \infty$ in a complex valued M-metric space (X, m). Then*

$$\lim_{n\to\infty}(m(x_n, y) - m_{x_n y}) = m(x, y) - m_{xy},$$

for all $y \in X$.

Lemma 4.5. *Assume that $x_n \to x$ and $x_n \to y$ as $n \to \infty$ in a complex valued M-metric space (X, m). Then $m(x, y) = m_{xy}$. Further, if $m(x, x) = m(y, y)$, then $x = y$.*

Proof. By Lemma 4.3, we have

$$0 = \lim_{n\to\infty}(m(x_n, x_n) - m_{x_n x_n}) = m(x, y) - m_{xy}.$$

□

5. Main results

Let $S = \{z \in \mathbb{C} : 0 \precsim z\}$. Let Ψ denote the class of the functions $\psi : S \to S$ satisfying the following conditions:

(a) ψ is continuous;

(b) $\psi(t) \succ 0$ iff $t \succ 0$ and $\psi(0) = 0$.

conditions:

Theorem 5.1. *Let (X, m) be a complete complex valued M-metric space and $T : X \to X$ be a self-mapping satisfying*

$$\psi(m(Tx, Ty)) \precsim F\Big(\psi(m(x, y)), \phi(m(x, y))\Big) \quad \text{for all} \quad x, y \in X, \tag{3}$$

where $\psi, \phi \in \Psi$ and $F \in \mathcal{C}$. Then T has a unique fixed point.

Proof. Fix $x_0 \in X$ and define $x_n = T^n x_0$ for every $n = 1, 2, 3, \cdots$. We shall prove that

$$m(x_n, x_{n+1}) \to 0 \quad \text{as} \quad n \to \infty.$$

We have

$$\begin{aligned}\psi(m(x_n, x_{n+1})) &= \psi(m(Tx_{n-1}, Tx_n))\\ &\precsim F\Big(\psi(m(x_{n-1}, x_n)), \phi(m(x_{n-1}, x_n))\Big)\\ &\precsim \psi(m(x_{n-1}, x_n)). \end{aligned} \tag{4}$$

So we get

$$\psi(m(x_n, x_{n+1})) \precsim \psi(m(x_{n-1}, x_n)).$$

The fact that ψ is nondecreasing implies that the sequence $\{m(x_n, x_{n+1})\}$ is monotone decreasing in $\mathbb{C}$. So there is an $0 \precsim t \in \mathbb{C}$ such that

$$m(x_n, x_{n+1}) \to t \quad \text{as} \quad n \to \infty.$$

Letting $n \to \infty$ in (4), by definition of F and continuity of ψ and ϕ, we obtain

$$\psi(t) \precsim F\Big(\psi(t), \phi(t)\Big) \precsim \psi(t).$$

Thus $F\Big(\psi(t),\phi(t)\Big)=\psi(t)$, so $\psi(t)=0$ or $\phi(t)=0$, hence $t=0$, that is,

$$m(x_n,x_{n+1})\to 0 \quad \text{as} \quad n\to\infty.$$

Now, we want to show that $\{x_n\}$ is a m-Cauchy sequence in (X,m), so by Lemma 4.2, we will prove that $\{x_n\}$ is a Cauchy sequence in the complex valued metric space (X,m^w).
Recall that

(1) $\lim_{n\to\infty} m(x_n,x_{n+1})=0$;
(2) $0\precsim m_{x_nx_{n+1}}\precsim m(x_n,x_{n+1})\Rightarrow \lim_{n\to\infty} m_{x_nx_{n+1}}=0$;
(3) $m_{x_nx_{n+1}}=\min\{m(x_n,x_n),m(x_{n+1},x_{n+1})\}\Rightarrow \lim_{n\to\infty} m(x_n,x_n)=0$.

On the other hand

$$m_{x_nx_m}=\min\{m(x_n,x_n),m(x_m,x_m)\}\Rightarrow \lim_{n,m\to\infty} m_{x_nx_m}=0,$$

and

$$M_{x_nx_m}=\max\{m(x_n,x_n),m(x_m,x_m)\}\Rightarrow \lim_{n,m\to\infty} M_{x_nx_m}=0.$$

Assume that $\{x_n\}$ is not Cauchy in (X,m^w). Then there exist $0\prec\epsilon\in\mathbb{C}$ and subsequences $\{x_{l_k}\}$, $\{x_{n_k}\}$ of $\{x_n\}$ with $n_k>l_k>k$ such that

$$m^w(x_{l_k},x_{n_k})\succsim\epsilon.$$

Now, corresponding to l_k, we can choose n_k such that it is the smallest integer with $n_k>l_k$ and satisfying above inequality. Hence

$$m^w(x_{l_k},x_{n_k-1})\prec\epsilon.$$

So we have

$$\epsilon\precsim m^w(x_{l_k},x_{n_k})\precsim m^w(x_{l_k},x_{n_k-1})+m^w(x_{n_k-1},x_{n_k})\prec\epsilon+m^w(x_{n_k-1},x_{n_k}). \tag{5}$$

We know that

$$m^w(x_{n_k-1},x_{n_k})=m(x_{n_k-1},x_{n_k})-2m_{x_{n_k-1},x_{n_k}}+M_{x_{n_k-1},x_{n_k}}. \tag{6}$$

Now, by (5) and (6), we have

$$\lim_{k\to\infty} m^w(x_{n_k-1},x_{n_k})=0 \quad \text{and} \quad \lim_{k\to\infty} m^w(x_{l_k},x_{n_k})=\epsilon. \tag{7}$$

Again,

$$m^w(x_{n_k},x_{l_k})\precsim m^w(x_{n_k},x_{n_k-1})+m^w(x_{n_k-1},x_{l_k-1})+m^w(x_{l_k-1},x_{l_k}), \tag{8}$$

and

$$m^w(x_{n_k-1},x_{l_k-1})\precsim m^w(x_{n_k-1},x_{n_k})+m^w(x_{n_k},x_{l_k})+m^w(x_{l_k},x_{l_k-1}). \tag{9}$$

Letting $k\to\infty$ in (8) and (9), we get

$$\lim_{k\to\infty} m^w(x_{n_k-1},x_{l_k-1})=\epsilon.$$

Thus

$$\begin{aligned}\lim_{k\to\infty} m(x_{n_k-1},x_{l_k-1}) &= \lim_{k\to\infty}(m(x_{n_k-1},x_{n_k-1}-2m_{x_{n_k-1},x_{n_k-1}}+M_{x_{n_k-1},x_{n_k-1}})\\ &= \lim_{k\to\infty} m^w(x_{n_k-1},x_{n_k-1})=\epsilon.\end{aligned}$$

Now, by (3), we have

$$\begin{aligned}\psi(\epsilon) &= \lim_{k\to\infty}\psi(m(x_{n_k},x_{l_k}))\\ &\precsim \lim_{k\to\infty} F\Big(\psi(m(x_{n_k-1},x_{l_k-1})),\phi(m(x_{n_k-1},x_{l_k-1}))\Big).\end{aligned}$$

Therefore

$$\begin{aligned}\psi(\epsilon) &\precsim F\Big(\psi(\epsilon),\phi(\epsilon)\Big)\\ &\precsim \psi(\epsilon).\end{aligned}$$

This implies that $\psi(\epsilon) = 0$ or $\phi(\epsilon) = 0$, so $\epsilon = 0$, which is a contradiction. Thus $\{x_n\}$ is a Cauchy sequence in the complete complex valued metric space (X, m^w), and so $\{x_n\}$ is m-Cauchy in the complete complex valued M-metric space (X, m). Hence there exists some $v \in X$ such that

$$\lim_{n\to\infty}(m(x_n, v) - m_{x_n v}) = 0.$$

But, we have $\lim_{n\to\infty} m_{x_n v} = 0$, hence $\lim_{n\to\infty} m(x_n, v) = 0$. By Remark 3.2, $m(v, v) = 0$. Now, we want to show that v is a fixed point of T. By (3), we have

$$0 \precsim \psi(m(Tv,Tv)) \precsim F\Big(\psi(m(v,v)),\phi(m(v,v))\Big) = F\Big(\psi(0),\phi(0)\Big) = 0.$$

Hence

$$\psi(m(Tv,Tv)) = 0 \Rightarrow m(Tv,Tv) = 0.$$

On the other hand

$$\psi(m(x_n,Tv)) \precsim F\Big(\psi(m(x_{n-1},v)),\phi(m(x_{n-1},v))\Big).$$

Then letting $n \to \infty$ and making use of Lemma 4.4 together with the continuity of functions F, ψ and ϕ, we have

$$m(v,Tv) = 0.$$

Hence

$$m(v,v) = m(Tv,Tv) = m(v,Tv) = 0,$$

so by (cm1), we have $Tv = v$. Now, let $u, v \in X$ such that $Tu = u$ and $Tv = v$. Assume that $m(v, v) > 0$. By (3)

$$\begin{aligned}\psi(m(v,v)) = \psi(m(Tv,Tv)) &\precsim F\Big(\psi(m(v,v)),\phi(m(v,v))\Big)\\ &\precsim \psi(m(v,v)).\end{aligned}$$

Consequently, $\psi(m(v, v)) = 0$ or $\phi(m(v, v)) = 0$, thus $m(v, v) = 0$, which is a contradiction. Thus, $m(v, v) = 0$. Similarly, we find $m(u, u) = 0$. On the other hand, if $m(v, u) > 0$, then by (3), we have

$$\begin{aligned}\psi(m(v,u)) = \psi(m(Tv,Tu)) &\precsim F\Big(\psi(m(v,u)),\phi(m(v,u))\Big)\\ &\precsim \psi(m(v,u)).\end{aligned}$$

We deduce that $\psi(m(v, u)) = 0$ or $\phi(m(v, u)) = 0$, i.e., $m(v, u) = 0$. Thus

$$m(v,v) = m(u,u) = m(v,u) = 0.$$

By (cm1), we get $u = v$. □

Taking $F(s, t) = s - t$ where $s, t \in S = \{z \in \mathbb{C} : 0 \precsim z\}$, in Theorem 5.1, we get the following corollary.

Corollary 5.2. *Let (X, m) be a complete complex valued M-metric space and $T : X \to X$ be a self-mapping satisfying*

$$\psi(m(Tx, Ty)) \precsim \psi(m(x, y)) - \phi(m(x, y)) \quad \text{for all} \quad x, y \in X, \tag{10}$$

where $\psi, \phi \in \Psi$. Then T has a unique fixed point.

Remark 5.3. *If in Corollary 5.2, we restrict the set of complex numbers to set of real numbers and take $S = \mathbb{R}^+$, then we obtain Theorem 3.1 of* [12].

Remark 5.4. *If in Corollary 5.2, we restrict the set of complex numbers to set of real numbers and take $S = \mathbb{R}^+$, and $\phi(t) = (1-k)\psi(t)$ where $0 < k < 1$, then we obtain the M-metric generalization of the result in* [7].

Remark 5.5. *If in Corollary 5.2, we restrict the set of complex numbers to set of real numbers and take $S = \mathbb{R}^+$ and $\psi(t) = t$, then we obtain the M-metric generalization for the weakly contractive fixed point theorem in* [15].

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