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INTUITIONISTIC FUZZY I-CONVERGENT DIFFERENCE SEQUENCE SPACES DEFINED BY MODULUS FUNCTION

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Abstract. In this study, we introduce the intuitionistic fuzzy I-convergent difference sequence spaces ${I_\Delta}^{(\mu,\upsilon)}(f)$ and ${I^0_\Delta}^{(\mu,\upsilon)}(f)$ defined by modulus function. Also we establish a new topological space and investigate some topological properties in intuitionistic fuzzy I-convergent difference sequence spaces ${I_\Delta}^{(\mu,\upsilon)}$ and ${I^0_\Delta}^{(\mu,\upsilon)}$ defined by modulus function.

1. Introduction

Fuzzy set theory introduced by Zadeh [1] has been applied on different fields of mathematics such as in the theory of functions [2] and in the approximation theory [3]. Fuzzy topology plays an essential role in fuzzy theory. It deals with such conditions where the classical theories break down. The intuitionistic fuzzy normed space and intuitionistic fuzzy n-normed space which were investigated in [4]-[5] are the most contemporary improvements in fuzzy topology. Recently, the notion of I-convergence in intuitionistic fuzzy zweier I-convergent sequence spaces and intuitionistic fuzzy zweier I-convergent double sequence spaces have been introduced in [10]-[13] and [27]-[29].

The notion of statistical convergence was given by Steinhaus [14] and Fast [15] using the definition of density of the set of natural numbers. Many years later, statistical convergence was discussed by many researchers in the theory of fourier analysis, ergodic theory and number theory. Some statistical convergence types in intuitionistic fuzzy normed spaces and intuitionistic fuzzy n-normed spaces were studied in [6]-[9] and [26]. As an extended definition of statistical convergence and definition of I-convergence was introduced by Kostyrko, Salat and Wilczynski [16] by using the idea of I of subsets of the set of natural numbers. Recently, I- and I^*- convergence of double sequences have been studied by Das et al. [17].

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Some new sequence spaces were introduced by means of various matrix transformations in [21]-[23] and [30]-[32]. Kızmaz [20] defined the difference sequence spaces with the difference matrix as follows:

$$X(\Delta)=\{x=(x_k):\Delta x\in X\}$$

for $X=l_\infty,\ c,\ c_0$, where $\Delta x_k=x_k-x_{k+1}$ and Δ denotes the difference matrix $\Delta=(\Delta_{nk})$ defined by

$$\Delta_{nk}=\begin{cases}(-1)^{n-k}, & \text{if } n\le k\le n+1,\\ 0, & \text{if } 0\le k<n.\end{cases}$$

In this paper, we introduce the intuitionistic fuzzy I-convergent difference sequence spaces $I_\Delta{}^{(\mu,\upsilon)}(f)$ and $I_\Delta^0{}^{(\mu,\upsilon)}(f)$ defined by modulus function and investigate some topological properties of these new spaces.

2. Basic definitions

In this section, we give some definitions and notations which will be used for this study.

Definition 2.1. ([18]) A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a*1=a$ for all $a\in[0,1]$,
(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for each $a,\ b,\ c,d\in[0,1]$.

Definition 2.2. ([18]) A binary operation $\circ:[0,1]\times[0,1]\to[0,1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

(i) $\circ$ is associative and commutative,
(ii) $\circ$ is continuous,
(iii) $a\circ 0=a$ for all $a\in[0,1]$,
(iv) $a\circ b\le c\circ d$ whenever $a\le c$ and $b\le d$ for each $a,\ b,\ c,d\in[0,1]$.

Definition 2.3. ([4]) The five-tuple $(X,\ \mu,\upsilon,*,\circ)$ is said to be intuitionistic fuzzy normed linear space (or shortly IFNLS) is where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, μ, υ are fuzzy sets on $X\times(0,\infty)$, μ denotes the degree of membership and υ denotes the degree of nonmembership of $(x,t)\in X\times(0,\infty)$ satisfying the following conditions for every $x,y\in X$ and $s,t>0$:

(i) $\mu(x,t)+\upsilon(x,t)\le 1$,
(ii) $\mu(x,t)>0$,
(iii) $\mu(x,t)=1$ if and only if $x=0$,
(iv) $\mu(\alpha x,t)=\mu\left(x,\frac{t}{|\alpha|}\right)$ if $\alpha\ne 0$,
(v) $\mu(x,t)*\mu(y,s)\le\mu(x+y,t+s)$,
(vi) $\mu(x,.):(0,\infty)\to[0,1]$ is continuous,
(vii) $\lim_{t\to\infty}\mu(x,t)=1$ and $\lim_{t\to 0}\mu(x,t)=0$,
(viii) $\upsilon(x,t)<1$,
(ix) $\upsilon(x,t)=0$ if and only if $x=0$,

(x) $\upsilon(\alpha x, t) = \upsilon\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,
(xi) $\upsilon(x,t) \circ \upsilon(y,s) \geq \upsilon(x+y, s+t)$,
(xii) $\upsilon(x,.) : (0,\infty) \to [0,1]$ is continuous,
(xiii) $\lim_{t\to\infty} \upsilon(x,t) = 0$ and $\lim_{t\to 0} \upsilon(x,t) = 1$.

In this case (μ, υ) is called intuitionistic fuzzy linear norm.

Example 2.1. ([4]) Let$(X, \|.\|)$ be a normed linear space, and let $a * b = ab$ and $a \circ b = \min\{a+b, 1\}$ for all $a, b \in [0,1]$. For all $x \in X$ and every $t > 0$, consider

$$\mu(x,t) := \frac{t}{t+\|x\|} \text{ and } \upsilon(x,t) := \frac{\|x\|}{t+\|x\|}.$$

Then $(X, \mu, \upsilon, *, \circ)$ is an IFNLS.

Definition 2.4. ([4]) Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is convergent to $L \in X$ with respect to the intuitionistic fuzzy linear norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $\mu(x_k - L, t) > 1 - \varepsilon$ and $\upsilon(x_k - L, t) < \varepsilon$ for all $k \geq k_0$ where $k \in \mathbb{N}$. It is denoted by $(\mu, \upsilon) - \lim x = L$.

Theorem 2.1. *([19])Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. Then, a sequence $x = (x_k)$ in X is convergent to $L \in X$ if and only if $\lim_{k\to\infty} \mu(x_k - L, t) = 1$ and $\lim_{k\to\infty} \upsilon(x_k - L, t) = 0$.*

Definition 2.5.([16]) If X is a non-empty set, then a family of sets $I \subset P(X)$ is called an ideal in X if and only if

(i) $\emptyset \in I$,
(ii) for each $A, B \in I$ implies that $A \cup B \in I$, and
(iii) for each $A \in I$ and $B \subset A$ we have $B \in I$,

where $P(X)$ is the power set of X.

Definition 2.6.([16]) If X is a non-empty set, then a non-empty family of sets $F \subset P(X)$ is called a filter on X if and only if

(i) $\emptyset \notin F$,
(ii) for each $A, B \in F$ implies that $A \cap B \in F$, and
(iii) for each $A \in F$ and $B \supset A$, we have $B \in F$.

An ideal I is called non-trivial if $I \neq \emptyset$ and $X \notin I$. A non-trivial ideal $I \subset P(X)$ is called an admissible ideal in X if and only $\{\{x\} : x \in X\} \subseteq I$.

A relation between the concepts of an ideal and a filter is given by the following proposition.

Proposition 2.1. ([16]) Let $I \subset P(X)$ be a non-trivial ideal. Then the class $F = F(I) = \{M \subset X : M = X - A$, for some $A \in I\}$ is a filter on X. $F = F(I)$ is called the filter associated with the ideal I.

Definition 2.7. ([24]) Let $I \subset P(\mathbb{N})$ be a non-trivial ideal and $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. Then a sequence $x = (x_k)$ in X is said to be I-convergent to $L \in X$

with respect to the intuitionistic fuzzy linear norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$, the set

$$\{k \in \mathbb{N} : \mu(x_k - L, t) \leq 1 - \varepsilon \text{ or } \upsilon(x_k - L, t) \geq \varepsilon\} \in I.$$

In this case, we write $I^{(\mu,\upsilon)} - \lim x = L$.

Definition 2.8. ([25]) A function $f : [0, \infty) \to [0, \infty)$ is called a modulus function if

(i) $f(t) = 0$ if and only if $t = 0$,
(ii) $f(t + u) \leq f(t) + f(u)$,
(iii) f is nondecreasing, and
(iv) f is continuous from the right at zero.

3. Main results

In this paper, we defined a variant of ideal convergent sequence spaces called intuitionistic fuzzy ideal difference convergent sequence spaces defined by modulus function and investigated some topological properties of these spaces.

Let w be the space of all real sequences. Intuitionistic fuzzy I-convergent difference sequence spaces defined by modulus function are defined as

$$I_{\Delta}^{(\mu,\upsilon)}(f) = \{(x_k) \in w : \{k \in \mathbb{N} : f(\mu(\Delta x_k - L, t)) \leq 1 - \varepsilon \text{ or } f(\upsilon(\Delta x_k - L, t)) \geq \varepsilon\} \in I\}$$

and

$$I_{\Delta}^{0\,(\mu,\upsilon)}(f) = \{(x_k) \in w : \{k \in \mathbb{N} : f(\mu(\Delta x_k, t)) \leq 1 - \varepsilon \text{ or } f(\upsilon(\Delta x_k, t)) \geq \varepsilon\} \in I\}.$$

Moreover, an open ball $B_x(r,t)(f)$ with centre $x \in I_{\Delta}^{(\mu,\upsilon)}(f)$ and radius $r \in (0,1)$ with respect to t, is defined as follows:

$$B_x(r,t)(f) = \{y \in I_{\Delta}^{(\mu,\upsilon)}(f) : \{k \in \mathbb{N} : f(\mu(\Delta x_k - \Delta y_k, t)) \leq 1 - r \text{ or } f(\upsilon(\Delta x_k - \Delta y_k, t)) \geq r\} \in I\}.$$

Theorem 3.1. *$I_{\Delta}^{(\mu,\upsilon)}(f)$ and $I_{\Delta}^{0\,(\mu,\upsilon)}(f)$ are linear spaces.*

Proof. We prove the result for $I_{\Delta}^{(\mu,\upsilon)}(f)$. Similarly, it can be proved for $I_{\Delta}^{0\,(\mu,\upsilon)}(f)$. Let $(x_k), (y_k) \in I_{\Delta}^{(\mu,\upsilon)}$ and α, β be scalars. The proof is trivial for $\alpha = 0$ and $\beta = 0$. Let $\alpha \neq 0$ and $\beta \neq 0$. For a given $\varepsilon > 0$, choose $s > 0$ such that $(1-\varepsilon) * (1-\varepsilon) > 1 - s$ and $\varepsilon \circ \varepsilon < s$. Hence, we have

$$A_1 = \{k \in \mathbb{N} : f(\mu(\Delta x_k - L_1, t/2|\alpha|)) \leq 1 - \varepsilon \text{ or } f(\upsilon(\Delta x_k - L_1, t/2|\alpha|)) \geq \varepsilon\} \in I,$$

$$A_2 = \{k \in \mathbb{N} : f(\mu(\Delta y_k - L_2, t/2|\beta|)) \leq 1 - \varepsilon \text{ or } f(\upsilon(\Delta x_k - L_1, t/2|\beta|)) \geq \varepsilon\} \in I,$$

$$A_1^c = \{k \in \mathbb{N} : f(\mu(\Delta x_k - L_1, t/2|\alpha|)) > 1 - \varepsilon \text{ and } f(\upsilon(\Delta x_k - L_1, t/2|\alpha|)) < \varepsilon\} \in F(I),$$

and

$$A_2^c = \{k \in \mathbb{N} : f(\mu(\Delta y_k - L_2, t/2|\beta|)) > 1 - \varepsilon \text{ and } f(\upsilon(\Delta y_k - L_2, t/2|\beta|)) < \varepsilon\} \in F(I).$$

Let define the set $A_3 = A_1 \cup A_2$. Hence $A_3 \in I$. It follows that A_3^c is a non-empty set in $F(I)$. We will prove that for every $(x_k), (y_k) \in I_{\Delta}^{(\mu,\upsilon)}(f)$,

$$A_3^c \subset \{k \in \mathbb{N} : f(\mu((\alpha.\Delta x_k + \beta.\Delta y_k) - (\alpha L_1 + \beta L_2), t)) > 1 - s$$

and $f(\upsilon((\alpha.\Delta x_k+\beta.\Delta y_k)-(\alpha L_1+\beta L_2),t))<s\}$.

Let $m\in A_3^c$. In this case

$f(\mu(\Delta x_m-L_1,t/2|\alpha|))>1-\varepsilon$ and $f(\upsilon(\Delta x_m-L_1,t/2|\alpha|))<\varepsilon$

and

$f(\mu(\Delta y_m-L_2,t/2|\beta|))>1-\varepsilon$ and $f(\upsilon(\Delta y_m-L_2,t/2|\beta|))<\varepsilon$.

Then,

$$f(\mu((\alpha.\Delta x_m+\beta.\Delta y_m)-(\alpha L_1+\beta L_2),t))\geq f(\mu(\alpha.\Delta x_m-\alpha L_1,t/2))*f(\mu(\beta.\Delta y_m-\beta L_2,t/2))$$

$$=f(\mu(\Delta x_m-L_1,t/2|\alpha|))*f(\mu(\Delta y_m-L_2,t/2|\beta|))>(1-\varepsilon)*(1-\varepsilon)>1-s$$

and

$$f(\upsilon((\alpha.\Delta x_m+\beta.\Delta y_m)-(\alpha L_1+\beta L_2),t)\leq f(\upsilon(\alpha.\Delta x_m-\alpha L_1,t/2))\circ f(\upsilon(\beta.\Delta y_m-\beta L_2,t/2)))$$

$$=f(\upsilon(\Delta x_m-L_1,t/2|\alpha|))\circ f(\upsilon(\Delta y_m-L_2,t/2|\beta|))<\varepsilon\circ\varepsilon<s.$$

This proves that

$A_3^c\subset\{k\in\mathbb{N}:f(\mu((\alpha.\Delta x_k+\beta.\Delta y_k)-(\alpha L_1+\beta L_2),t))>1-s$

and $f(\upsilon((\alpha.\Delta x_k+\beta.\Delta y_k)-(\alpha L_1+\beta L_2),t))<s\}$.

Hence ${I_\Delta}^{(\mu,\upsilon)}(f)$ is a linear space.

Theorem 3.2. *Every closed ball $B_x^c(r,t)(f)$ is an open set in ${I_\Delta}^{(\mu,\upsilon)}(f)$.*

Proof. Let $B_x(r,t)(f)$ be an open ball with centre $x\in {I_\Delta}^{(\mu,\upsilon)}(f)$ and radius $r\in(0,1)$ with respect to t, i.e.

$B_x(r,t)(f)=\{y\in {I_\Delta}^{(\mu,\upsilon)}(f):\{k\in\mathbb{N}:f(\mu(\Delta x_k-\Delta y_k,t))\leq 1-r \text{ or } f(\upsilon(\Delta x_k-\Delta y_k,t))\geq r\}\in I\}$.

Let $y\in B_x^c(r,t)(f)$. Then $f(\mu(\Delta x-\Delta y,t))>1-r$ and $f(\upsilon(\Delta x-\Delta y,t))<r$.

Since $f(\mu(\Delta x-\Delta y,t))>1-r$, there exists $t_0\in(0,t)$ such that $f(\mu(\Delta x-\Delta y,t_0))>1-r$ and $f(\upsilon(\Delta x-\Delta y,t_0))<r$.

Let $r_0=f(\mu(\Delta x-\Delta y,t_0))$. Since $r_0>1-r$, there exists $s\in(0,1)$ such that $r_0>1-s>1-r$ and so there exists $r_1,r_2\in(0,1)$ such that $r_0*r_1>1-s$ and $(1-r_0)\circ(1-r_2)<s$.

Let $r_3=max\{r_1,r_2\}$. Then $1-s<r_0*r_1\leq r_0*r_3$ and $(1-r_0)\circ(1-r_3)\leq(1-r_0)\circ(1-r_2)<s$.

Consider the closed balls $B_y^c(1-r_3, t-t_0)(f)$ and $B_x^c(r,t)(f)$. We prove that $B_y^c(1-r_3, t-t_0)(f) \subset B_x^c(r,t)(f)$. Let $z \in B_y^c(1-r_3, t-t_0)(f)$. Then $f(\mu(\Delta y - \Delta z, t-t_0)) > r_3$ and $f(\upsilon(\Delta y - \Delta z, t-t_0)) < 1-r_3$. Hence

$f(\mu(\Delta x - \Delta z, t)) \geq f(\mu(\Delta x - \Delta y, t_0)) * f(\mu(\Delta y - \Delta z, t-t_0)) > r_0 * r_3 \geq r_0 * r_1 > 1-s > 1-r$ and

$f(\upsilon(\Delta x - \Delta z, t)) \leq f(\upsilon(\Delta x - \Delta y, t_0)) \circ f(\upsilon(\Delta y - \Delta z, t-t_0)) < (1-r_0) \circ (1-r_3) < s < r.$

Thus $z \in B_x^c(r,t)(f)$ and hence $B_y^c(1-r_3, t-t_0)(f) \subset B_x^c(r,t)(f)$.

Remark. *It is clear that* ${I_\Delta}^{(\mu,\upsilon)}(f)$ *is an IFNLS. Define*

$$
{\tau_\Delta}^{(\mu,\upsilon)}(f) = \left\{ A \subset {I_\Delta}^{(\mu,\upsilon)}(f) : for\, each\, x \in A \;\, there\, exist\, t>0 \;\, and \;\, r \in (0,1) \; such \;\, that \;\, B_x^c(r,t)(f) \subset A \right\}.
$$

Then ${\tau_\Delta}^{(\mu,\upsilon)}(f)$ *is a topology on* ${I_\Delta}^{(\mu,\upsilon)}(f)$.

Theorem 3.3. *The topology* ${\tau_\Delta}^{(\mu,\upsilon)}(f)$ *on* ${I_\Delta^0}^{(\mu,\upsilon)}(f)$ *is first countable.*

Proof. It is clear that $\{B_x^c(\frac{1}{n}, \frac{1}{n})(f) : n \in \mathbb{N}\}$ is a local base at $x \in {I_\Delta}^{(\mu,\upsilon)}(f)$. Then the topology ${\tau_\Delta}^{(\mu,\upsilon)}(f)$ on ${I_\Delta^0}^{(\mu,\upsilon)}(f)$ is first countable.

Theorem 3.4. ${I_\Delta}^{(\mu,\upsilon)}(f)$ *and* ${I_\Delta^0}^{(\mu,\upsilon)}(f)$ *are Hausdorff spaces.*

Proof. Let $x, y \in {I_\Delta}^{(\mu,\upsilon)}(f)$ such that $x \neq y$. Then $0 < f(\mu(\Delta x - \Delta y, t)) < 1$ and $0 < f(\upsilon(\Delta x - \Delta y, t)) < 1$.

Let define r_1, r_2 and r such that $r_1 = f(\mu(\Delta x - \Delta y, t))$, $r_2 = f(\upsilon(\Delta x - \Delta y, t))$ and $r = max\{r_1, 1-r_2\}$. Then for each $r_0 \in (r,1)$ there exist r_3 and r_4 such that $r_3 * r_4 \geq r_0$ and $(1-r_3) \circ (1-r_4) \leq (1-r_0)$.

Let $r_5 = max\{r_3, (1-r_4)\}$ and consider the closed balls $B_x^c(1-r_5, \frac{t}{2})(f)$ and $B_y^c(1-r_5, \frac{t}{2})(f)$. Then clearly $B_x^c(1-r_5, \frac{t}{2})(f) \cap B_y^c(1-r_5, \frac{t}{2})(f) = \emptyset$

Suppose that $z \in B_x^c(1-r_5, \frac{t}{2})(f) \cap B_y^c(1-r_5, \frac{t}{2})(f)$, then

$r_1 = f(\mu(\Delta x - \Delta y, t)) \geq f(\mu\left(\Delta x - \Delta z, \frac{t}{2}\right)) * f(\mu\left(\Delta y - \Delta z, \frac{t}{2}\right)) \geq r_5 * r_5 \geq r_3 * r_4 \geq r_0 > r$

and

$r_2 = f(\upsilon(\Delta x - \Delta y, t)) \leq f(\upsilon\left(\Delta x - \Delta z, \frac{t}{2}\right)) \circ f(\upsilon\left(\Delta y - \Delta z, \frac{t}{2}\right) \leq (1-r_5)) \circ (1-r_5) \leq (1-r_3) \circ (1-r_4) \leq (1-r_0) < 1-r,$

which is a contradiction. Hence ${I_\Delta}^{(\mu,\upsilon)}(f)$ is a Hausdorff space.

Theorem 3.5. *Let ${I_\Delta}^{(\mu,\upsilon)}(f)$ be an IFNLS, ${\tau_\Delta}^{(\mu,\upsilon)}(f)$ be a topology on ${I_\Delta}^{(\mu,\upsilon)}(f)$ and (x_k) be a sequence in ${I_\Delta}^{(\mu,\upsilon)}(f)$. Then a sequence (x_k) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ,υ) if and only if $f(\mu(\Delta x_k - \Delta x_0, t)) \longrightarrow 1$ and $f(\upsilon(\Delta x_k - \Delta x_0, t)) \longrightarrow 0$ as $k \longrightarrow \infty$.*

Proof. Let $B_{x_0}(r,t)(f)$ be an open ball with centre $x_0 \in {I_\Delta}^{(\mu,\upsilon)}(f)$ and radius $r \in (0,1)$ with respect to t, i.e.

$$B_{x_0}(r,t)(f) = \{(x_k) \in {I_\Delta}^{(\mu,\upsilon)}(f) : \{k \in \mathbb{N} : f(\mu(\Delta x_k - \Delta x_0, t)) \leq 1 - r \text{ or } f(\upsilon(\Delta x_k - \Delta x_0, t)) \geq r\} \in I\}.$$

Suppose that a sequence (x_k) in ${I_\Delta}^{(\mu,\upsilon)}(f)$ is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ,υ). Then for $r \in (0,1)$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $(x_k) \in B^c_{x_0}(r,t)(f)$ for all $k \geq k_0$. Thus

$$\{k \in \mathbb{N} : f(\mu(\Delta x_k - \Delta x_0, t)) > 1 - r \text{ and } f(\upsilon(\Delta x_k - \Delta x_0, t)) < r\} \in F(I).$$

So $1 - f(\mu(\Delta x_k - \Delta x_0, t)) < r$ and $f(\upsilon(\Delta x_k - \Delta x_0, t)) < r$, for all $k \geq k_0$. Then $f(\mu(\Delta x_k - \Delta x_0, t)) \longrightarrow 1$ and $f(\upsilon(\Delta x_k - \Delta x_0, t)) \longrightarrow 0$ as $k \longrightarrow \infty$.

Conversely, suppose that for each $t > 0$, $f(\mu(\Delta x_k - \Delta x_0, t)) \longrightarrow 1$ and $f(\upsilon(\Delta x_k - \Delta x_0, t)) \longrightarrow 0$ as $k \longrightarrow \infty$. Then for $r \in (0,1)$, there exists $k_0 \in \mathbb{N}$ such that $1 - f(\mu(\Delta x_k - \Delta x_0, t)) < r$ and $f(\upsilon(\Delta x_k - \Delta x_0, t)) < r$ for all $k \geq k_0$. So, $f(\mu(\Delta x_k - \Delta x_0, t)) > 1 - r$ and $f(\upsilon(\Delta x_k - \Delta x_0, t)) < r$ for all $k \geq k_0$. Hence $(x_k) \in B^c_{x_0}(r,t)(f)$ for all $k \geq k_0$. This proves that a sequence (x_k) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ,υ).

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