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A NOTE ON THE RELATIVE GROWTH INDICATORS OF DIFFERENTIAL POLYNOMIALS GENERATED BY ENTIRE AND MEROMORPHIC FUNCTIONS

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Abstract. In the paper we establish the relationship between the relative L-order (relative L^*-order), relative L-type (relative L^*-type) and relative L-weak type (relative L^*-weak type) of a meromorphic function f with respect to an entire function g and that of differential polynomial generated by meromorphic f and entire g.

1. Introduction, Definitions and Notations

We denote by $\mathbb{C}$ the set of all finite complex numbers. Let f be a non-constant meromorphic function defined in the open complex plane $\mathbb{C}$. Also let n_{0j}, n_{1j},......,$n_{kj}\,(k \geq 1)$ be non-negative integers such that for each j, $\sum\limits_{i=0}^{k} n_{ij} \geq 1$. We call $M_j\,[f] = A_j\,(f)^{n_{0j}}\left(f^{(1)}\right)^{n_{1j}}$........$\left(f^{(k)}\right)^{n_{kj}}$ where $T\,(r, A_j) = S\,(r, f)$ to be a differential monomial generated by f. The numbers $\gamma_{M_j} = \sum\limits_{i=0}^{k} n_{ij}$ and $\Gamma_{M_j} = \sum\limits_{i=0}^{k} (i+1)\,n_{ij}$ are called respectively the degree and weight of $M_j\,[f]$ $\{[1],[7]\}$. The expression $P\,[f] = \sum\limits_{j=1}^{s} M_j\,[f]$ is called a differential polynomial generated by f. The numbers $\gamma_P = \max\limits_{1<j<s} \gamma_{M_j}$ and $\Gamma_P = \max\limits_{1<j<s} \Gamma_{M_j}$ are called respectively the degree and weight of $P\,[f]$ $\{[1],[7]\}$. Also we call the numbers $\underline{\gamma}_P = \min\limits_{1<j<s} \gamma_{M_j}$ and k (the order of the highest derivative of f) the lower degree and the order of $P\,[f]$ respectively. If $\underline{\gamma}_P = \gamma_P$, $P\,[f]$ is called a homogeneous differential polynomial. Throughout the paper we consider only the non-constant differential polynomials and we denote by $P_0\,[f]$ a differential polynomial not containing f i.e. for which $n_{0j} = 0$ for $j = 1, 2, ..., s$. We consider only those $P\,[f]\,, P_0\,[f]$ singularities of whose individual terms do not cancel each other.

The following two definitions are well known:

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Definition 1. *The quantity $\Theta(a;f)$ of a meromorphic function f is defined as follows*

$$\Theta(a;f) = 1 - \limsup_{r\to\infty} \frac{\overline{N}(r,a;f)}{T(r,f)} .$$

Definition 2. [6] *For $a \in \mathbb{C} \cup \{\infty\}$, let $n_p(r,a;f)$ denotes the number of zeros of $f-a$ in $|z| \leq r$, where a zero of multiplicity $< p$ is counted according to its multiplicity and a zero of multiplicity $\geqslant p$ is counted exactly p times; and $N_p(r,a;f)$ is defined in terms of $n_p(r,a;f)$ in the usual way. We define*

$$\delta_p(a;f) = 1 - \limsup_{r\to\infty} \frac{N_p(r,a;f)}{T(r,f)} .$$

The following definitions are also well known.

Definition 3. *The order ρ_f and lower order λ_f of a meromorphic function f are defined as*

$$\rho_f = \limsup_{r\to\infty} \frac{\log T_f(r)}{\log r} \text{ and } \lambda_f = \liminf_{r\to\infty} \frac{\log T_f(r)}{\log r} .$$

Somasundaram and Thamizharasi [8] introduced the notions of L-order and L-lower order for entire functions where $L \equiv L(r)$ is a positive continuous function increasing slowly i.e., $L(ar) \sim L(r)$ as $r \to \infty$ for every positive constant "a". Their definitions are as follows:

Definition 4. [8] *The L-order ρ_f^L and the L-lower order λ_f^L of a meromorphic function f are defined as follows:*

$$\rho_f^L = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log[rL(r)]} \text{ and } \lambda_f^L = \liminf_{r\to\infty} \frac{\log T(r,f)}{\log[rL(r)]} .$$

The more generalised concept of L-order and L-lower order of meromorphic functions are L^*-order and L^*-lower order respectively which are as follows:

Definition 5. *The L^*-order $\rho_f^{L^*}$ and the L^*-lower order $\lambda_f^{L^*}$ of a meromorphic function f are defined by*

$$\rho_f^{L^*} = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log\left[re^{L(r)}\right]} \text{ and } \lambda_f^{L^*} = \liminf_{r\to\infty} \frac{\log T(r,f)}{\log\left[re^{L(r)}\right]} .$$

For an entire function g, the Nevanlinna's characteristic function $T_g(r) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ \left|g(re^{i\theta})\right| d\theta$ where $\log^+ x = \max(0, \log x)$ for $x > 0$.

If g is non-constant then $T_g(r)$ is strictly increasing and continuous and its inverse $T_g^{-1} : (T_g(0), \infty) \to (0, \infty)$ exists and is such that $\lim_{s\to\infty} T_g^{-1}(s) = \infty$.

Lahiri and Banerjee [5] introduced the definition of relative order of a meromorphic function with respect to an entire function which is as follows:

Definition 6. [5] *Let f be meromorphic and g be entire. The relative order of f with respect to g denoted by $\rho_g(f)$ is defined as*

$$\begin{aligned} \rho_g(f) &= \inf\{\mu > 0 : T_f(r) < T_g(r^\mu) \text{ for all sufficiently large } r\} \\ &= \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} . \end{aligned}$$

The definition coincides with the classical one [5] if $g(z) = \exp z$.

Similarly one can define the relative lower order of a meromorphic function f with respect to an entire g denoted by $\lambda_g(f)$ in the following manner :

$$\lambda_g(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} .$$

Datta and Biswas [2] gave the definition of relative type and relative weak type of a meromorphic function with respect to an entire function g which are as follows:

Definition 7. [2] *The relative type $\sigma_g(f)$ of a meromorphic function f with respect to an entire function g are defined as*

$$\sigma_g(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}}, \quad \text{where } 0 < \rho_g(f) < \infty .$$

Similarly one can define the lower relative type $\overline{\sigma}_g(f)$ in the following way

$$\overline{\sigma}_g(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}}, \quad \text{where } 0 < \rho_g(f) < \infty .$$

Definition 8. [2] *The relative weak type $\tau_g(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative lower order $\lambda_g(f)$ is defined by*

$$\tau_g(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\lambda_g(f)}} .$$

Analogously one can define the growth indicator $\overline{\tau}_g(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative lower order $\lambda_g(f)$ as

$$\overline{\tau}_g(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\lambda_g(f)}} .$$

In order to prove our results we require the following definitions:

Definition 9. *The relative L-order $\rho_g^L(f)$ and the relative L-lower order $\lambda_g^L(f)$ of a meromorphic function f with respect to an entire function g are defined as follows:*

$$\rho_g^L(f) = \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log [rL(r)]} \text{ and } \lambda_g^L(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log [rL(r)]} .$$

Definition 10. *The relative L-type $\sigma_g^L(f)$ and the relative L-lower type $\overline{\sigma}_g^L(f)$ of a meromorphic function f with respect to an entire function g are defined as follows:*

$$\sigma_g^L(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{[rL(r)]^{\rho_g^L(f)}} \text{ and } \overline{\sigma}_g^L(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{[rL(r)]^{\rho_g^L(f)}},$$

where $0 < \rho_g^L(f) < \infty$.

Definition 11. *The relative L-weak type $\tau_g^L(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative L-lower order $\lambda_g^L(f)$ is defined by*

$$\tau_g^L(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{[rL(r)]^{\lambda_g^L(f)}} .$$

Similarly one can define the growth indicator $\overline{\tau}_g^L(f)$ *of a meromorphic function* f *with respect to an entire function* g *with finite positive relative* L*-lower order* $\lambda_g^L(f)$ *as*

$$\overline{\tau}_g^L(f) = \limsup_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\lambda_g^L(f)}}.$$

The more generalised concept of relative L-order (relative L-lower order), relative L-type (relative L-lower type) and relative L-weak type of meromorphic function with respect to an entire function are relative L^*-order (relative L^*-lower order), relative L^*-type (relative relative L^*-lower type) and relative L^*-weak type respectively which are as follows:

Definition 12. *The* L^**-order* $\rho_f^{L^*}$ *and the* L^**-lower order* $\lambda_f^{L^*}$ *of a meromorphic function* f *are defined by*

$$\rho_g^{L^*}(f) = \limsup_{r\to\infty} \frac{\log T_g^{-1}T_f(r)}{\log\left[re^{L(r)}\right]} \text{ and } \lambda_g^{L^*}(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1}T_f(r)}{\log\left[re^{L(r)}\right]}\ .$$

Definition 13. *The relative* L^**-type* $\sigma_g^{L^*}(f)$ *and the relative* L^**-lower type* $\overline{\sigma}_g^{L^*}(f)$ *of a meromorphic function* f *with respect to an entire function* g *are defined as follows:*

$$\sigma_g^{L^*}(f) = \limsup_{r\to\infty} \frac{T_g^{-1}T_f(r)}{\left[re^{L(r)}\right]^{\rho_g^{L^*}(f)}} \text{ and } \overline{\sigma}_g^{L^*}(f) = \liminf_{r\to\infty} \frac{T_g^{-1}T_f(r)}{\left[re^{L(r)}\right]^{\rho_g^{L^*}(f)}},$$

where $0 < \rho_g^{L^*}(f) < \infty$.

Definition 14. *The relative* L^**-weak type* $\tau_g^{L^*}(f)$ *of a meromorphic function* f *with respect to an entire function* g *with finite positive relative* L^**-lower order* $\lambda_g^{L^*}(f)$ *is defined by*

$$\tau_g^{L^*}(f) = \liminf_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\lambda_g^{L^*}(f)}}\ .$$

Similarly one can define the growth indicator $\overline{\tau}_g^{L^*}(f)$ *of a meromorphic function* f *with respect to an entire function* g *with finite positive relative* L^**-lower order* $\lambda_g^{L^*}(f)$ *as*

$$\overline{\tau}_g^{L^*}(f) = \limsup_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\lambda_g^{L^*}(f)}}\ .$$

In this paper we wish to establish the relationship between the relative L-order (relative L^*-order), relative L-type (relative L^*-type) and relative L-weak type (relative L^*-weak type) of a meromorphic function f with respect to an entire function g and that of polynomial generated by the meromorphic f and entire g. We use the standard notations and definitions in the theory of entire and meromorphic functions which are available in [4] and [9].

2. Lemmas

In this section we present two lemmas which will be needed in the sequel.

Lemma 1. [3] *Let f be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum\limits_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum\limits_{a \neq \infty} \delta(a; f) = 1$ and g be an entire function with regular growth and non zero finite order. Also let $\Theta(\infty; g) = \sum\limits_{a \neq \infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum\limits_{a \neq \infty} \delta(a; g) = 1$. Then for homogeneous $P_0[f]$ and $P_0[g]$,*

$$\lim_{r \to \infty} \frac{\log T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{\log T^{-1}_g T_f(r)} = 1 \ .$$

Lemma 2. [3] *Let f be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum\limits_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum\limits_{a \neq \infty} \delta(a; f) = 1$ and g be an entire function with regular growth and non zero finite type. Also let $\Theta(\infty; g) = \sum\limits_{a \neq \infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum\limits_{a \neq \infty} \delta(a; g) = 1$. Then for homogeneous $P_0[f]$ and $P_0[g]$,*

$$\lim_{r \to \infty} \frac{T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{T^{-1}_g T_f(r)} = \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \ .$$

3. Theorems

In this section we present the main results of the paper.

Theorem 1. *Let f be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum\limits_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum\limits_{a \neq \infty} \delta(a; f) = 1$ and g be an entire function with regular growth having non zero finite order and $\Theta(\infty; g) = \sum\limits_{a \neq \infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum\limits_{a \neq \infty} \delta(a; g) = 1$. Then the relative L-order and relative L-lower order order of $P_0[f]$ with respect to $P_0[g]$ are same as those of f with respect to g for homogeneous $P_0[f]$ and $P_0[g]$.*

Proof. By Lemma 1 we obtain that

$$\begin{aligned}
\rho^L_{P_0[g]}(P_0[f]) &= \limsup_{r \to \infty} \frac{\log T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{\log [rL(r)]} \\
&= \limsup_{r \to \infty} \left\{ \frac{\log T^{-1}_g T_f(r)}{\log [rL(r)]} \cdot \frac{\log T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{\log T^{-1}_g T_f(r)} \right\} \\
&= \limsup_{r \to \infty} \frac{\log T^{-1}_g T_f(r)}{\log [rL(r)]} \cdot \lim_{r \to \infty} \frac{\log T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{\log T^{-1}_g T_f(r)} \\
&= \rho^L_g(f) \cdot 1 \\
&= \rho^L_g(f) \ .
\end{aligned}$$

In a similar manner, $\lambda^L_{P_0[g]}(P_0[f]) = \lambda^L_g(f)$.
This proves the theorem. □

Theorem 2. *Let f be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum\limits_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum\limits_{a \neq \infty} \delta(a; f) = 1$ and g be an entire function with regular growth having non zero finite order and*

$\Theta(\infty; g) = \sum_{a \neq \infty} \delta_p(a; g) = 1$ *or* $\delta(\infty; g) = \sum_{a \neq \infty} \delta(a; g) = 1$. *Then the relative* L^*-*order and relative* L^*-*lower order of* $P_0[f]$ *with respect to* $P_0[g]$ *are same as those of* f *with respect to* g *i.e.,*

$$\rho^{L^*}_{P_0[g]}(P_0[f]) = \rho^{L^*}_g(f) \text{ and } \lambda^{L^*}_{P_0[g]}(P_0[f]) = \lambda^{L^*}_g(f)$$

where $P_0[f]$ *and* $P_0[g]$ *are homogeneous.*

We omit the proof of Theorem 2 because it can be carried out in the line of Theorem 1.

Theorem 3. *Let* f *be a meromorphic function either of finite order or of non-zero lower order such that* $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ *or* $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$ *and* g *be an entire function of regular growth having non zero finite type and* $\Theta(\infty; g) = \sum_{a \neq \infty} \delta_p(a; g) = 1$ *or* $\delta(\infty; g) = \sum_{a \neq \infty} \delta(a; g) = 1$. *Then the relative* L-*type and relative* L-*lower type of* $P_0[f]$ *with respect to* $P_0[g]$ *are* $\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}}$ *times that of* f *with respect to* g *if* $\rho^L_g(f)$ *is positive finite where* $P_0[f]$ *and* $P_0[g]$ *are homogeneous.*

Proof. From Lemma 2 and Theorem 1 we get that

$$\begin{aligned}
\sigma^L_{P_0[g]}(P_0[f]) &= \limsup_{r \to \infty} \frac{T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{[rL(r)]^{\rho_{P_0[g]}(P_0[f])}} \\
&= \lim_{r \to \infty} \frac{T^{-1}_{P_0[g]} T_{P_0[f]}(r)}{T^{-1}_g T_f(r)} \cdot \limsup_{r \to \infty} \frac{T^{-1}_g T_f(r)}{[rL(r)]^{\rho^L_g(f)}} \\
&= \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \sigma^L_g(f) \ .
\end{aligned}$$

Similarly $\overline{\sigma}^L_{P_0[g]}(P_0[f]) = \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \cdot \overline{\sigma}^L_g(f)$.
Thus the theorem is established. □

Theorem 4. *Let* f *be a meromorphic function either of finite order or of non-zero lower order such that* $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ *or* $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$ *and* g *be an entire function of regular growth having non zero finite type and* $\Theta(\infty; g) = \sum_{a \neq \infty} \delta_p(a; g) = 1$ *or* $\delta(\infty; g) = \sum_{a \neq \infty} \delta(a; g) = 1$. *Then the relative* L^*-*type and relative* L^*-*lower type of* $P_0[f]$ *with respect to* $P_0[g]$ *are* $\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}}$ *times that of* f *with respect to* g *if* $\rho^{L^*}_g(f)$ *is positive finite when* $P_0[f]$ *and* $P_0[g]$ *are homogeneous.*

We omit the proof of Theorem 4 because it can be carried out in the line of Theorem 3 and with the help of Theorem 2.

Similarly one may state the following two theorems without their proofs because those can be carried out in the line of Theorem 3 and Theorem 4 respectively.

Theorem 5. *Let f be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$ and g be an entire function of regular growth having non zero finite type and $\Theta(\infty; g) = \sum_{a \neq \infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum_{a \neq \infty} \delta(a; g) = 1$. Then $\tau^L_{P_0[g]}(P_0[f])$ and $\overline{\tau}^L_{P_0[g]}(P_0[f])$ are $\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}}$ times that of f with respect to g i.e.,*

$$\tau^L_{P_0[g]}(P_0[f]) = \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \cdot \tau^L_g(f) \text{ and } \overline{\tau}^L_{P_0[g]}(P_0[f]) = \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \cdot \overline{\tau}^L_g(f)$$

when $\lambda^L_g(f)$ is positive finite and $P_0[f]$, $P_0[g]$ are homogeneous.

Theorem 6. *If f be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$ and g be an entire function of regular growth having non zero finite type and $\Theta(\infty; g) = \sum_{a \neq \infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum_{a \neq \infty} \delta(a; g) = 1$, then $\tau^{L^*}_{P_0[g]}(P_0[f])$ and $\overline{\tau}^{L^*}_{P_0[g]}(P_0[f])$ are $\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}}$ times that of f with respect to g i.e.,*

$$\tau^{L^*}_{P_0[g]}(P_0[f]) = \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \cdot \tau^{L^*}_g(f) \text{ and } \overline{\tau}^{L^*}_{P_0[g]}(P_0[f]) = \left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}} \cdot \overline{\tau}^{L^*}_g(f)$$

when $\lambda^{L^*}_g(f)$ is positive finite and $P_0[f]$, $P_0[g]$ are homogeneous.

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