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EXAMINING LONG-TERM MEMORY AND ASYMMETRY IN LOG RETURNS OF THE TOP40 AND NSE20 INDICES

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Abstract. In this research study, we explore the existence of long-term memory in return patterns within the financial markets of South Africa and Kenya during the time frame spanning 1995 to 2010. Our empirical findings reveal notable linear autocorrelation of third order for the NSE20 index and first order autocorrelation for the TOP40 index.Furthermore, we reveal strong evidence of changing variance within both indices, along with increased autocorrelation in absolute returns for both market contexts. Theoretical autocorrelation functions are fitted, and parameters are estimated. Various ARCH-type models conditioned on a normal distribution are examined. Among these models, the A-PGARCH model, based on absolute daily returns $|y_t|^d, d \in (1, 2)$, notably outperforms four other models (TGARCH, GARCH, GARCH-M, and GJR-GARCH) in modeling the evolving variance and volatility asymmetry in the two emerging markets.

1. Introduction

Autoregressive conditional heteroscedasticity models (hereafter ARCH) introduced by [1] and later extended to generalized ARCH model by [2] have been successfully applied in financial time series. GARCH models conditioned on normal distribution, have been very popular, and effective for modeling the volatility dynamics in many markets for example [3] investigated presence of stylized facts across African equity indices. In vast empirical finance literature, such models within GARCH framework conditioned on non-normal probability densities such as student t, NIG, variance gamma etc, have been suggested and extensively analyzed. Sample autocorrelations of log returns $y_t = \ln S_t - \ln S_{t-1}$ of stock indices S_t are assumed to be tiny as opposed to the sample autocorrelations of the absolute and squared values which are known to be significantly different from zero see [5]. This behavior suggests that there is some kind of long-range dependence in the data. Moreover, [6] postulated that latent news process do have two different components, commonly referred to as normal news and unusual events. The first component has a mean zero, with a normal stochastic forcing process, and

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the jump innovation is the second component. The two components are assumed to be contemporaneously independent. In this study we follow [5] to investigate presence of long memory property of the two indices for the first component of y_t conditioned on normal with standardized residues being not only leptokurtic but also identically and independently distributed in general.

In this article, we investigate a long memory property for both indexes, by studying absolute returns $|y_t|^d, d \in (0.5, 6)$. The autocorrelation of y_t and $|y_t|^d,\ d > 0$, is examined. There is a lot of literature on modeling and forecasting volatility, see for example [4], which is slightly different to what we tend to investigate.

The remainder of the article is arranged as in the following order: Section 2 present the data description and time plots of log returns. In section 3 we carry out linear autocorrelation analysis of $|y_t|^d$ for changing values of d and fit a theoretical autocorrelation function for the two indices. Parameter estimates of different GARCH type models are presented in section 4. Section 5 draws the conclusion of the analysis.

2. Data description

We analyze secondary daily data for TOP40 index and NSE20 index dated July 03,1995 to September 02,2010 and July 03,1995 to April 22,2010 respectively. A sample size of $n = 3651$ observations. **R** Statistical software, is used to carry out the empirical analysis. Let S_t denote the value of the value underlying process at time $t = 1, ..., 3651$ and $y_t = \log S_t - \log S_{t-1}$,denote daily log returns. It follows quite easily that

$$\begin{aligned} y_t &= \log\left(\frac{S_t}{S_{t-1}}|\mathcal{G}_{t-1}\right) = \mu + \sigma_t Z_t, \\ \text{where} \quad & Z_t \sim i.i.d.N(0,1),\ \sigma_t \in \text{ARCH models},\ \text{and } \mathcal{G}_{t-1} \text{ is a filtration set.} \end{aligned}$$

Empirical interrogation of stylized facts of log returns begins with the descriptive statistics, and time plots as shown in the Table 1.

Table 1. Descriptive statistics of daily log returns y_t

	sample(T)	mean	median	var	std	skew	kurt
NSE20	3651	0.000051	-0.000076	0.000072	0.008461	0.445422	8.096728
TOP40	3651	0.000449	0.000887	0.000212	0.014571	-0.422583	6.145587

2.1. Basic statistics and timeplots.

We observe from Table1 that the kurtosis of y_t is higher than 3 from both indices. The Jarque-Bera normality test statistic for the two indices is above the set critical value for a normal distribution. In general, this suggests a characteristic of non-normal or leptokurtic property of daily log returns. Time plots of absolute log returns in Figure 1, indicates that there are general nonlinear trend inherent in returns. This implies that the market volatility is changing over time, and ARCH type model would be suitable for modeling a time varying volatility in both markets.

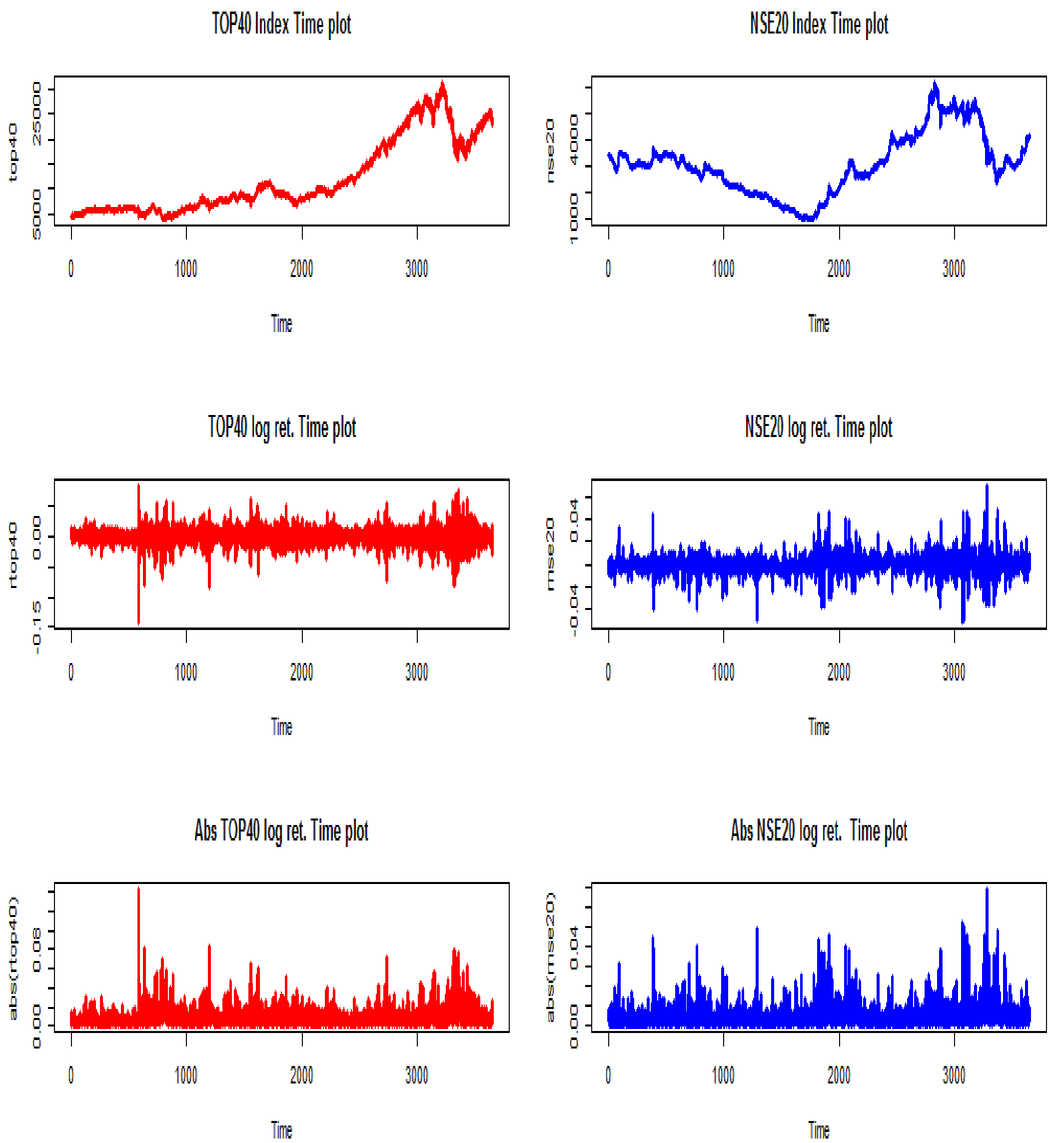

FIGURE 1. TOP40 and NSE20 index time-plot, daily log returns and absolute daily log returns,1995 − 2010

3. METHODOLOGY

3.1. **GARCH models.** The article analyzes the volatility structure of TOP40 index and NSE20 index. We note that previous studies, show that modeling asymmetric components, is more crucial than specifying error distributions for improving volatility forecasts of financial returns in the presence of fat-tailed, leptokurtic, skewed innovations and leverage effects. If asymmetries are neglected, the GARCH models conditioned with normal distribution is preferable to those models with sophisticated error distribution, see [3] for example. The model used in this study incorporates various volatility models including ARMA and asymmetric GARCH models conditioned on normal distribution. If a series $\{y_t\}$ follows an ARMA(p,q) model then $\{y_t\}$ can be described as follows,

$$y_t = \mu + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t,\ \ \varepsilon_t \sim N(0,\sigma_t^2)$$

where $a_i,\ b_j\ \varepsilon_j, i=1,...,p, j=1,...,q$ are parameters. GARCH model of [2] can be expressed as ARCH(∞) using a backshift operator $\mathbb{L}$ such that

$$\sigma_t^2 = \omega + \alpha(\mathbb{L})\varepsilon_t^2 + \beta(\mathbb{L})\sigma_t^2$$

with $\alpha(\mathbb{L}) = \alpha_1\mathbb{L} + \alpha_2\mathbb{L}^2 + ... + \alpha_q\mathbb{L}^q$ and $\beta(\mathbb{L}) = \beta_1\mathbb{L} + \beta_2\mathbb{L} + ... + \beta_p\mathbb{L}^p$. If all the roots lie outside the unit circle, we have

$$\begin{aligned}\sigma_t^2 &= \frac{\omega}{1-\beta(\mathbb{L})} + \frac{\alpha(\mathbb{L})}{1-\beta(\mathbb{L})}\varepsilon_t \\ &= \frac{\omega}{1-\beta_1-\beta_2-...-\beta_p} + \sum_{j=1}^{\infty}\phi_j\varepsilon_{t-j}^2\end{aligned}$$

where $\varepsilon_t|\sigma_t \sim N(0,\sigma_t^2)$ which is ARCH(∞) process, introduced by [1].

The assumption of error distribution is the main reason as to why we use the maximum likelihood estimation (MLE) to the model. Suppose that the error term is driven by a normal distribution, the uncorrelated standardized residuals are observed to be leptokurtic in nature in tandem with what is in literature about the stylized facts of financial time series log returns data. Our main interest is to study the long term changing variance of log returns in emerging markets. The log-likelihood function is given by

$$Ł_T = -\frac{1}{2}\sum_{t=1}^{T}\left(\ln(2\pi) + \ln(\sigma_t^2) + z_t^2\right)$$

where z_t is independently and identically distributed normal $(i.i.d.N(0,1))$ and T is the number of observations.

3.2. **Specifications of conditional variance.** More formally, to model asymmetry as well as long memory property of the conditional variance process, let

$$y_t = \mu + \varepsilon_t;\ \varepsilon_t = \sigma_t z_t,\ z_t \sim N(0,1)$$

where σ_t^2 can take any of the following functional forms among others

$$\begin{aligned}
\text{ARCH(q) } \sigma_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \\
\text{GARCH(p,q) } \sigma_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \\
\text{TGARCH(p,q) , } \sigma_t^2 &= \omega + \sum_{i=1}^{q} \left[\alpha_i |\varepsilon_{t-i}| + \gamma_i |\varepsilon_{t-i}^{+}|\right] + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \\
\text{GJR-GARCH(p,q) , } \sigma_t^2 &= \omega + \sum_{i=1}^{q} \left[\alpha_i + \gamma_i \mathcal{I}_{\varepsilon_{t-i}>0}\right] \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \\
\text{A-PGARCH(p,q) , } \sigma_t^\delta &= \omega + \sum_{i=1}^{q} \alpha_i \left[|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}\right]^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta .
\end{aligned}$$

Empirical studies by [7] hereafter (GJR-GARCH model) show that it is crucial to include asymmetric term γ in financial log return models. See [8] for more empirical studies. Asymmetry is estimated in GJR-GARCH model including a dummy variable in the conditional variance function which takes the value "1" for negative shocks and "0" otherwise. The conditional variance including the dummy variable is defined as TGARCH model of [9]. In general, for A-PGARCH(1,1) specification, we estimate the parameters of the model,

$$\begin{aligned}
y_t &= \mu + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_t^2) \\
\sigma_t^\delta &= \omega + \alpha(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^\delta + \beta \sigma_{t-1}^\delta \\
\alpha + \beta &< 1;\ \omega > 0, \delta > 0
\end{aligned}$$

where γ reflect the leverage effect and $\delta > 0$ is the Box-Cox transformation of σ_t. In general,this model nests other models, such as

- ARCH Engle(1982) when $\delta = 2, \gamma = 0, \beta = 0$
- GARCH Bolerslev(1986) when $\delta = 2, \gamma = 0$
- GJR Glosten etal(1993) when $\delta = 2$

We fit ARMA(k,0) APARCH(1,1) model conditioned on a $i.i.d.(0,1)$ distribution.

4. Linear Autocorrelation

We examine the sample autocorrelations for the two indices. Sample autocorrelations of the absolute returns $|y_t|^d$ for various positive values of d were investigated. Figure 3 gives $corr(|y_t|^d, |y_{t+\tau}|^d)$ for $d = 0.125, 0.25, 0.5, 0.75, ..., 3$ at lag (τ) 1 to 5, and 20 ,40 80,100.We note that both market returns have similar shape and the optimal correlation is attained when $d \in (1, 3)$.

4.1. **Sample Autocorrelation Curve.**

As per the study by [5], the autocorrelation $\rho_\tau(d)$ is observed to be a smooth function of d across a range of values, with d taking on values such as $0.0125, 0.25, ..., 3$, and τ ranging from 1 to 230. Our own findings from the two datasets align with this observation.

Empirical evidence suggests the presence of either a saddle point or an optimal point denoted as $\hat{d}$ within the range of (2,3). When $d < \hat{d}$, $\rho_\tau(d)$ behaves as a concave function of d, and when $d > \hat{d}$, it takes on a convex shape concerning d.

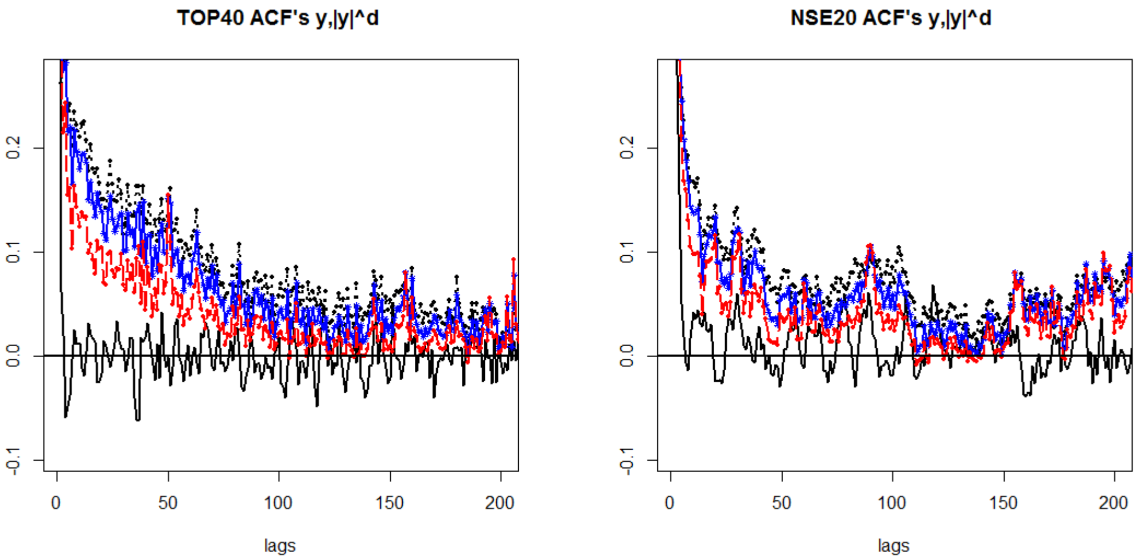

Figure 2. ACF plots of log returns(continuous black near zero), absolute log returns(decaying exponentially dotted black) $d = 1$, when $d = 1.45$ (blue), and squared log returns respectively $d = 2$(red)

Table 2 presents the lags at which the first negative τ^* autocorrelations of $|r_\tau|^d$ occur for both indices.

Table 2. Lags(τ^*) at which first negative autocorrelations of $|y_t|^d$ occurs at various d

d	1.1125	1.4875	1.8625	2.9875	3.2375	3.3625	3.8625	5.1125
TOP40 τ^*	235	232	185	76	43	40	38	7
NSE20 τ^*	281	280	111	44	44	44	14	11

We fit the preferred theoretical autocorrelation function (see [5]) specified as model characterised by the graphical representation in Figure 2 and Figure 4.

$$\rho_\tau = \frac{\alpha\rho_{\tau-1}^{\beta_1}\beta_2^\tau}{\tau^{\beta_3}}; \qquad \Rightarrow \log\rho_\tau = \log\alpha + \beta_1\log\rho_{\tau-1} + \tau\log\beta_2 - \beta_3\log\tau$$

$$\Rightarrow \log\rho_\tau = \alpha^* + \beta_1^*\log\rho_{t-1} + \beta_2^*\tau + \beta_3^*\log\tau$$

The fitted model for the two indices (TOP40 and NSE20) are as given below. The parameter $\alpha^* = \log_e\alpha$ is not significant for both cases suggesting that $\alpha = 1$ with the adjusted coefficient of determination being more than 70% as shown in Table 3, hence

$$\text{TOP40 } \rho_\tau = \frac{\rho_{\tau-1}^{0.187}0.9965^\tau}{\tau^{0.397}}, \text{ NSE20 } \rho_\tau = \frac{\rho_{\tau-1}^{0.7223}1.00189^\tau}{\tau^{0.2864}}.$$

Moreover the ACF of the absolute returns $|y_t|^d$, $d = 1.4875$ for all sets portray an exponential decay with up to more than 180 lags as shown in Figure 4. We note

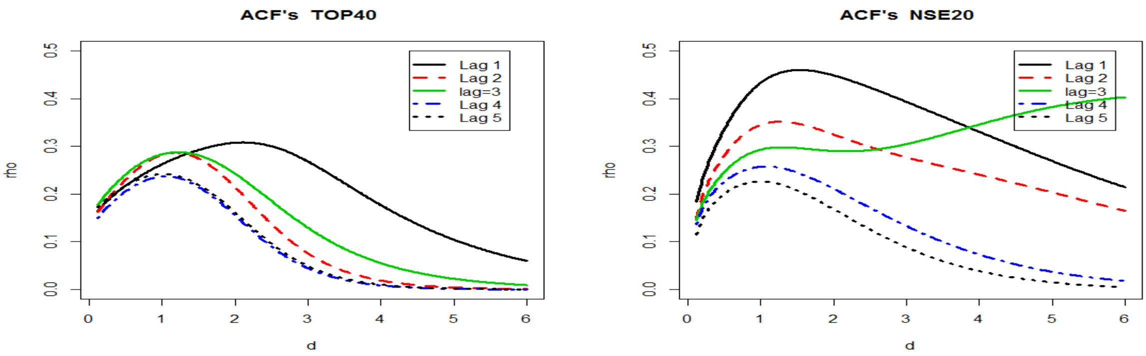

FIGURE 3. $corr(|y_t|^d, |y_{t-h}|^d)$ for $h = 1, 2, 3, 4, 5$ and $d \in (0.125, 6)$

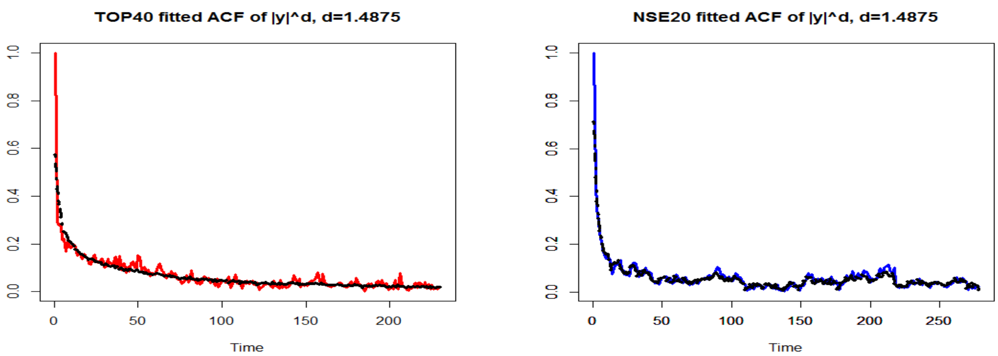

FIGURE 4. ACF of $|y|^d$ at lags $3, 5, 10, 18$ and 25

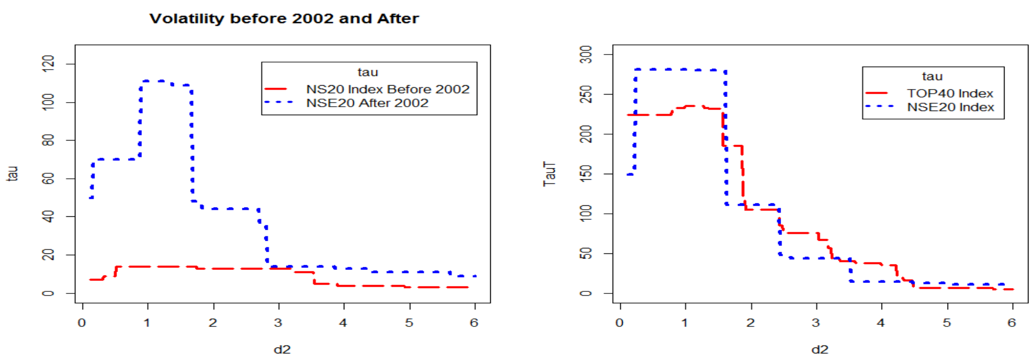

FIGURE 5. Lags(τ^*) at which positive autocorrelation of $|y_t|^d$ occurs for various values of d for both indices. First negative autocorrelation $|y_t|^d$ for NSE20 and TOP40 index two periods, for various values of d before year 2002 and after.

TABLE 3. Ordinary least square estimates of autocorrelation curve for TOP40 and NSE20

TOP40	Estimate	Std. Error	t value	Pr(>|t|)
α^*	-0.321421	0.217052	-1.480849	0.140040
β_1^*	0.186969	0.065238	2.865946	0.004550
β_2^*	-0.003561	0.001039	-3.428513	0.000721
β_3^*	-0.397235	0.072170	-5.504180	0.000000
R^2	72.36%			
NSE20	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
α^*	0.216095	0.176980	1.221016	0.223130
β_1^*	0.722323	0.039845	18.128172	0.000000
β_2^*	0.001893	0.000654	2.895598	0.004089
β_3^*	-0.286406	0.059772	-4.791607	0.000003
R^2	72.04%			

that all the power transformations of the absolute returns have a significant positive correlations at least up to lag 150 which support the claim that both indexes have a long term memory.

4.2. **Sensitivity of Autocorrelation structure.** It should be noted from NSE20 time plot of log returns the volatility structure differs considerably before September 9, 2002 and after as seen in Figure 1, of NSE20 time plot. It is of interest to look at the memory structure for those two periods,i.e, *July 3,1995-September 18,2002* and *September 19,2002-April 22,2010*. From the two periods, their volatility structure

TABLE 4. NSE20 index lags at which first negative autocorrelation $|y_t|^d$ occurs for the two periods, before September 2002,and after September 2002, $d = 0.125 : 0.0125 : 6$

d	0.7375	1.3625	1.4875	1.9875	3.2375	3.3625	5.1125	5.9875
Before 2002	14	14	14	13	13	11	3	3
After 2002	70	111	109	44	14	14	11	9

was computed and captured in form of a plot as shown in Figure 5 for both cases. It should be noted that the volatility structure differed considerably. The latest period appeared more volatile since the autocorrelations for $|y_t|^d$ were higher compared to autocorrelations of the period before September 2002.

5. EMPIRICAL RESULTS

5.1. **GARCH Modeling Framework.** Different ARCH type models were calibrated, GARCH(1,1), GJR-GARCH(1,1), A-PGARCH(1,1), TGARCH, and GARCH-M. All models parameters were calibrated for TOP40 returns and NSE20 returns conditioned on normal distribution and the following results were obtained.

(1a) AR(1)-GARCH(1,1) model (*TOP40 Index*)

$$y_t = \mu + \rho_1 y_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N(0, s_t^2),\ s_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta s_{t-1}^2$$

Table 5. AR(1)-GARCH(1,1) parameter estimates TOP40 daily log returns

	AR-GARCH	model	$\mathfrak{L}$=10851.65	
TOP40	Estimate	Std. Error	t value	Pr(>|t|)
μ	0.000738385	0.000175565	4.205752296	0.000026022
ρ_1	0.084743371	0.017654094	4.800210861	0.000001585
ω	0.000002611	0.000000644	4.052084305	0.000050763
α	0.112672243	0.011034056	10.211317320	0.000000000
β	0.879740256	0.011310857	77.778392863	0.000000000

(1b) AR(3)-GARCH(1,1) model (*NSE20 Index*)

$$y_t = \mu + \rho_1 y_{t-l} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + \varepsilon_t,\ \varepsilon_t \sim N(0, s_t^2);\ s_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta s_{t-1}^2$$

Table 6. AR(3)-GARCH(1,1) parameter estimates NSE20 daily log returns

	AR-GARCH	model	$\mathfrak{L}$=13100.03	
NSE20	Estimate	Std. Error	t value	Pr(>|t|)
ρ_1	0.231799948	0.019157966	12.099402834	0.000000000
ρ_2	0.148208534	0.018867985	7.855027075	0.000000000
ρ_3	0.085868572	0.018372337	4.673796946	0.000002957
ω	0.000002648	0.000000433	6.117165582	0.000000001
α	0.157141540	0.014737370	10.662793756	0.000000000
β	0.805228376	0.017568789	45.832890462	0.000000000

From the parameters estimated, the mean equation μ_t and the variance equation were found to be significant at 1% level of significance for both data sets. As expected the sum of the coefficients were less than one, in line with parameter constraints.

(2a) AR(1)-APARCH(1,1) model *TOP40 Index*

$$y_t = \mu + \rho_l y_{t-l} + \varepsilon_t,\ \varepsilon_t \sim N(0, s_t^2),\ s_t^\delta = \omega + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta s_{t-1}^\delta$$

Table 7. AR(1)-A-PGARCH(1,1) parameter estimates TOP40 index demeaned log returns

	AR-A-PGARCH	model	$\mathfrak{L}$=10882.45	
TOP40	Estimate	Std. Error	t value	Pr(>|t|)
μ	0.000446213	0.000180346	2.474207628	0.013353207
ρ	0.080278817	0.017703970	4.534509349	0.000005774
ω	0.000030027	0.000006429	4.670296083	0.000003008
α	0.103844502	0.010732953	9.675296537	0.000000000
γ	0.325353624	0.053181626	6.117782569	0.000000001
β	0.892995476	0.010132462	88.132132037	0.000000000
δ	1.480272914	0.162621382	9.102572470	0.000000000

Table 8. AR(3)-A-PGARCH(1,1) parameter estimates NSE20 index daily log returns

	AR-A-PGARCH	model	$\mathfrak{L}$=13110.36	
NSE20	Estimate	Std. Error	t value	Pr(>|t|)
ρ_1	0.226269257	0.019044635	11.880997511	0.000000000
ρ_2	0.147387126	0.018570537	7.936610879	0.000000000
ρ_3	0.086177369	0.018178677	4.740574176	0.000002131
ω	0.000018443	0.000002941	6.272196128	0.000000000
α	0.157952998	0.013339433	11.841057583	0.000000000
γ	-0.089074505	0.034001156	-2.619749316	0.008799443
β	0.825642118	0.016221437	50.898210315	0.000000000
δ	1.589747632	0.174237595	9.124021876	0.000000000

(2b) AR(3)-APARCH(1,1) *NSE20 Index*

$$y_t = \mu+\rho_l y_{t-l}+\rho_2 y_{t-2}+\rho_3 y_{t-3}+\varepsilon_t,\ \varepsilon_t \sim N(0, s_t^2);\ s_t^\delta = \omega+\alpha(|\varepsilon_{t-1}|-\gamma\varepsilon_{t-1})^\delta+\beta s_{t-1}^\delta$$

The parameter δ for both indexes compares favourably with the value of $d = 1.4875$ investigated earlier under the absolute returns $|y_t|^d$ as reported in Figure 5.

5.2. **Discussions.** As a general observation, both markets seem to have an autocorrelation model of order AR(1) for TOP40 and order three AR(3) for NSE20 index respectively. Conditioned on standard normal distribution, GARCH type model seem to be calibrated and the resulting standardized residual has no element of serial autocorrelation but heavy tailed $i.i.d.(0,1)$ as expected. All these results are summarized in Tables 5, 6, 7, 8 respectively. In order to determine whether

Table 9. Likelihood values for all models studied and their respective ranking based on χ^2 test.

Model	TOP40 $\mathfrak{L}$	Rank	χ^2	NSE20 $\mathfrak{L}$	RANK	χ^2
GARCH-M	10852.42	4	60.06	13102.13	3	16.46
GARCH	10851.65	5	61.60	13100.03	4	20.66
A-PGARCH	10882.45	1		13110.36	1	
TGARCH	10876.26	3	12.38	13097.18	5	32.36
GJR-GARCH	10876.32	2	12.26	13102.32	2	16.08

any of the four models are not the true underlying model at a given significance level, a statistical test is required. The classical maximum likelihood ratio test is used to differentiate between the various models. Let l_o denote the log likelihood values under the null hypothesis that the true model is one of the four models (GARCH-M, GARCH, TGARCH, and GJR-GARCH) estimated, and let l represent the log likelihood value under the alternative hypothesis that the true model is A-PGARCH. Consequently, $2(l - l_o)$ is expected to follow a χ^2 distribution with 2 degrees of freedom if the hypothesis holds true. The χ^2 test values in Table 9 significantly surpass the critical values at the 5% level or any other reasonable level. Therefore, we reject the idea that the data is generated by any of the other four models examined, in favor of the A-PGARCH model.

5.3. **Volatility asymmetry.** If investors exhibit slow reactions to price increases but tend to overreact during market declines, this behavior can exacerbate market risks. This phenomenon may be a contributing factor to the emergence of asymmetries in market volatility due to irrational investments. The asymmetry of the A-PGARCH model is captured by the parameter γ. In addition, the relative asymmetry introduced by [10] is calculated.

$$\text{volatility asymmetry} = \frac{(1+\gamma)}{(1-\gamma)^{\delta}}$$

This metric measures the extent to which the response of volatility to a negative shock exceeds its response to a positive shock. Volatility asymmetry was computed for both indices, yielding a value of 2.3733 for the TOP40 index and 0.79537 for the NSE20 index. These estimates closely resemble the findings reported in [11].

6. Conclusions

In both data sets, we found obvious clustered changing variance. In view of modeling long term volatility, the autocorrelations of $|y_t|^d,\ d > 0$ were investigated and the result compared with A-PGARCH δ parameter which seems to tally. This implies the presence of long term memory in both data sets. In this article, we explored the A-PGARCH model and some of its extensions in the context of modeling volatility of emerging markets. There is a strong evidence that both markets exhibits long memory and asymmetry as supported in literature. Different ARCH type models were compared against A-PGARCH model. Using log likelihood function estimate and the χ^2 test, all other ARCH models were rejected in favor of A-PGARCH model. The volatility structure for NSE20 index, differed considerably between the two periods (before September 2002, and after September 2002). The long term memory property that was found in NSE20 index was attributed to the "after September 2002 " period. The autocorrelations of $|y_t|^d$ gave the largest values and long lags of positive autocorrelations before becoming negative for the first time.

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