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MODELING STOCK RETURNS AND TRADING VOLUME IN REGIME SWITCHING WORLD

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Abstract. According to the available literature, there has not been much research into modeling the correlation between stock returns, volatility, and trading volume. Models such as GARCH and its extensions have been used to represent this relationship and reproduce the stylized realities of financial time series. However, empirical studies have demonstrated that the model fails to describe the volatility persistence of financial markets that exhibit regime shifts adequately. A model that permits the GARCH parameters to transition regimes in line with a Markov process is a solution to this problem. GARCH and Markov-Switching GARCH models are used in this study to model the link between stock returns, volatility, and trading volume of both developed and emerging stock market indices. The impact of including trade volume as an exogenous variable in the GARCH model's conditional variance equation on volatility persistence is also studied. For all of the indices returns and volume, the data provide evidence of common features of time series data such as clustering of volatility,leverage effects, and a distribution that is leptokurtic. As the data frequency changes from daily to weekly, the persistence of volatility is observed to decrease across the two market indices. The addition of trade volume in the conditional variance equation of GARCH (1,1) lead to reduced volatility persistence. Furthermore, the series are distinguished by the process of volatility remaining in the regime with low volatility longer than it does in the high volatility regime, and the volatility process differing across the two regimes.

1. Introduction

Previous financial modeling research in both developed and emerging stock market exchanges has not exhaustively looked into the relationship between stock returns, volatility, and volume. This relationship is significant because it enables a better understanding of the microstructure of financial markets. Volatility is a statistical measure of a security's or index's return dispersion that may be calculated using the variance of a return series from a market index. A higher standard deviation estimate indicates that the investment's returns are more dispersed and

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the risk is higher. The stock market's success is heavily reliant on volatility, and as volatility reduces, the stock market may rise or fall, and vice versa. This means that as volatility rises, risk rises as well, and returns may fall. On the other hand,trading volume is defined by Abbondante [1] as the total number of shares traded each day. Trading volume can be used to estimate stock value growth; in other words, trading volume is an important technical indicator that can be used to confirm a trend or a trend reversal. It gives investors an idea of how a security's price is moving and if they should buy or sell it. When trade volume increases, prices generally move in the same direction. In this way, we point out that in financial time series modeling, a thorough understanding of the relationship between stock returns, volatility, and volume is essential. According to Wiley & Daigler [20], the significance of information in determining prices is linked to the connection between price and volume. On the other hand, Karpoff [16] contends that the price-volume empirical relationship is crucial because it provides insight into the comprehension of the numerous guesses that compete to widely propagate thoughts about information flow into the market.

Furthermore, according to a research by Attari et al. [4],larger returns to investment stimulate investors, resulting in capital flow, however in a volatile environment, returns are not clear and investment forecasting is difficult. According to Attari et al. [4] and Glascock & Hsieh [12], developing securities exchanges are linked to extremely volatile stock returns due to low volume in the stock market. Girard & Baswas [11] reports a negative association between the two markets in their study that contrasted trading volume and stock returns volatility in the developed and emerging stock economies. Trading volume is well thought-out as a crucial technical indicator in gauging the strength of a market, according to Al Samman and Al-Jafari [2], because it incorporates information regarding stock performance. The dynamic and real-time link between trading volume and stock returns has been the subject of empirical studies. Returns granger-cause trading volume in developed markets, according to Lee & Rui [17]. DeMedeiros et al. [9] report a real-time and dynamic relationship between returns, volatility, and trading volume using Brazilian market data in their study. According to Mahajan & Singh [18], there is a positive relationship between trade volume and volatility. Furthermore, studies by Christie [7] show that volume and volatility have a negative association. Crouch [8] looked into the relationship between day-to-day trading volume, absolute stock market fluctuations, and individual equities and discovered a favorable link. Rogalski [19] established a positive contemporaneous relationship between trading volume and absolute returns using month-to-month stock data.

A recent study by Jiranyakul [15], which looked at the dynamic relationship between stock returns, transaction volume, and volatility on the Thai stock exchange, found that trade volume is important in dynamic relationships. When it comes to the subprime mortgage crisis in the United States, the volume of transaction creates both returns and volatility. Furthermore, the existing literature indicates that the Thai stock exchange has a contemporaneous relationship between trading volume and volatility. The GARCH model and its extensions have been used to represent stylized characteristics of financial market data, such as clustering of volatility, heavy tails, long-memory, skewed, and non-linear qualities, among others, based on the current literature. Furthermore, empirical research demonstrates that financial market volatility poses a number of issues that GARCH-type models fail to reflect well, see Bauwens [5]. These models, in particular, frequently exhibit a high level of conditional volatility persistence. Recent research shows that when there is a regime change in volatility dynamics, the GARCH-type model cannot reflect the actual change in volatility, see Ardia et al. [3]. Allowing the GARCH model parameters to change over time according to a latent Markov process is one solution to this challenge. This model, for example, favors volatility estimates that can respond fast to changes in unconditional volatility. The GARCH Markov-Switching model is the name of the method presented here. This model was developed by Hamilton [14] and is useful for time series research because it can capture more complicated dynamic patterns in stock data if transitions between regimes are allowed.

Despite the remarkable empirical and theoretical efforts on the phenomenon of stock returns, volatility, and volume relationships on numerous stock exchanges, blended results have been reported in general. The majority of the findings, on the other hand, confirmed that stock returns, volatility, and trading volume all had a contemporaneous relationship. While there is a lot of empirical evidence on the relationship between stock returns, volatility, and trading volume in developed and developing financial markets, there aren't many studies that look at the Nairobi Securities Exchange (NSE) (an emerging market) in comparison to developed stock markets. As a result, this study uses the GARCH and Markov Switching models to analyze the relationship between stock returns, volatility, and trading volume in developed and developing financial markets. The impact of introducing trading volume as an exogenous variable in GARCH on volatility persistence is also explored.

The rest of the article is structured as follows: Section 2 gives a summary of the GARCH and Markov-Switching GARCH models that were employed in the study. The data used, as well as descriptive statistics and a general discussion of the research findings, are presented in Section 3. Section 4 brings the document to a close.

2. Methodology

2.1. **Modeling the underlying asset.** Let the random process that describes the market uncertainty be defined by $(\Omega, \mathcal{F}_t, \mathbb{P})$ under probability measure $\mathbb{P}$ and where $\mathcal{F}_t$ is the information flow that is driven by a Brownian motion of the stochastic process. Denote the stock price at time t by P_t and let it be adapted to the natural filtration $\mathcal{F}_t$. Let R_t be the log returns defined by $R_t = \log_e\left(\frac{P_t}{P_{t-1}}\right)$, hence

$$\begin{array}{l} R_t = \log_e\left(\frac{P_t}{P_{t-1}}|\mathcal{F}_{t-1}\right), \quad \Longrightarrow \; R_t = \mu_t + r_t, \; \text{ where } r_t = \sigma_t\varepsilon_t \\ \sigma_t^2 = Var(R_t|\mathcal{F}_{t-1}) = \mathbb{E}\big[(R_t-\mu_t)^2|\mathcal{F}_{t-1}\big] \\ \varepsilon_t \;\sim\; i.i.d(0,1) \quad \Longrightarrow \quad \varepsilon_t|\mathcal{F}_{t-1} \;\sim\; \mathbb{D}(0,\sigma_t). \end{array} \tag{2.1}$$

Here $\mathbb{D}$ stands for the assumed distribution, whereas μ_t, σ_t and r_t are the conditional mean, conditional variance and the adjusted return on asset respectively.

2.2. **GARCH Models.** The GARCH model is a basic conceptual structure of Bollerslev [6] and it is the general case of the ARCH model which is a postulate of Engle [10]. GARCH models have the capability to capture both clustering of volatility and accounting for the changing variance of time series data. Define a

GARCH (p,q) model as

$$r_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \tag{2.2}$$

such that p and q are the ARCH and GARCH orders respectively and, ε_t is a sequence of i.i.d random variables with mean and variance zero and unity respectively. That is, $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$. These conditions imply that the unconditional variance of r_t is finite, whereas its conditional variance σ^2 evolves over time. This study utilizes GARCH(1, 1) model since it is the widely used model and expressed as follows;

$$r_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{2.3}$$

2.3. **Markov-Switching GARCH Model.** Suppose that the mean of r_t is zero and there is no serial correlation, that is, assume that $\mathbb{E}[r_t] = 0$ and $\mathbb{E}[r_t, r_{t-i}] = 0$ for $i \neq 0$ and all $t > 0$. It is realistic to make this assumption if one is dealing with high-frequency returns whose conditional mean is usually assumed to be zero. In this study conditional variance process is allowed to switch regimes and in addition, the observed information set until time $t-1$ is denoted by $\mathcal{F}_{t-1}$. Thus, the general case of a Markov-Switching GARCH model is written as: $r_t|(s_t, \mathcal{F}_{t-1}) \sim D(0, \sigma_{k,t}^2, \epsilon_t)$. In this equation, $D(0, \sigma_{kt}^2, \epsilon_t)$ represents a continuous distribution whose mean is zero, has a changing variance with time, $\sigma_{k,t}^2$ and an additional shape parameters ϵ_k collected in the vector. The random variable s_t defined in the discrete space $1, ..., K$ represents the Markov-switching GARCH model. We define the standardized innovation as $\eta_{k,t} = r_t/(\sigma_{k,t}) \sim D(0, 1, \epsilon_k)$. To model the dynamics of the state of random variables, it is assumed that the state s_t evolves according to an unobserved first order homogeneous Markov chain with a probability transition matrix of order KxK. In this case, we consider two states, that is, $K = 2$, so the transition probability matrix P is given by

$$P_{i,j} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

where $P_{i,j} = P[s_t = j | s_{t-1} = i]$ is the transition probability from state $s_{t-1} = i$ to $s_t = j$,$0 \leq P_{i,j} \leq 0 \ \forall \ i, j \in \{1, 2\}$ and that $\sum_{j=1}^{2} P_{i,j} = 1, \ \forall \ i \in \{1, 2\}$. Considering the parametrization of $\mathcal{D}(.)$, we've the variance of r_t^2 conditional on the realization of $s_t = k$, that is, $\sigma_{k,t}^2 = \mathbb{E}[r_t^2 | s_t = k, \mathcal{F}_{t-1}]$

2.3.1. *Conditional variance dynamics.* It is assumed that the conditional variance of r_t follow a GARCH-type model, see Haas et al. [13]. Conditionally on regime $s_t = k$, $\sigma_{k,t}^2$ is a function of the past observations, r_{t-1} , past variance, $\sigma_{k,t}^2$ and vector of parameters, θ_k which is regime-dependent, that is,

$$\sigma_{k,t}^2 = h(r_{t-1}, \sigma_{k,t-1}^2, \theta_k). \tag{2.4}$$

Here $h(.)$ is a $\mathcal{F}_{t-1}$-function that defines the conditional variance filter and ensures that it is positive. The initial variance recursions, that is, $\sigma_{k,t}^2 (k = 1, 2)$, are set equal to the unconditional variance in regime k. Depending on the form of $h(.)$, different scedastic specifications are obtained, and in our case we consider GARCH scedastic specifications as below. According to Bollerslev [6], the GARCH model is given by

$$\sigma_{k,t}^2 = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2 + \beta_k \sigma_{k,t-1}^2 \tag{2.5}$$

for $k = 1, ..., K$. We have $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \beta_k)'$ in this case and in order to ensure positivity, it demands that $\alpha_{0,k} > 0, \alpha_{1,k} \geq 0$ and $\beta_k \geq 0$. Moreover,covariance-stationarity is ensured in each regime by requiring that $(\alpha_{1,k} + \beta_k) < 1$.

2.3.2. *Model parameter estimation.* The maximum likelihood estimation technique is utilized to estimate the MSGARCH as reported by Ardia et al. [3]. Let vector $\Psi = (\theta_1, \epsilon_1, ..., \theta_K, \epsilon_K, \mathbb{P})$ represent the model parameters, then the likelihood function is given by

$$L(\Psi|\mathcal{F}_T) = \prod_{t=1}^{T} f(r_t|\Psi, \mathcal{F}_{t-1}) \tag{2.6}$$

where $f(r_t|\Psi, \mathcal{F}_{t-1})$ is the probability density function of r_t given the past observations, $\mathcal{F}_{t-1}$ and the parameters of the model Ψ. Therefore, the conditional density of r_t, for MSGARCH model is given by

$$f(r_t|\Psi, \mathcal{F}_{t-1}) = \sum_{i=1}^{K}\sum_{j=1}^{K} P_{ij} z_{i,t-1} f_D(r_t|s_t = j, \Psi, \mathcal{F}_{t-1}) \tag{2.7}$$

where $z_{i,t-1} = P[s_{t-1} = j|\Psi, \mathcal{F}_{t-1}]$ is the filtered probability of state i at time $t-1$ obtained via Hamilton filter, see Hamilton [14]. In equation (2.7), the density of r_t in state $s_t = k$ given Ψ and $\mathcal{F}_{t-1}$ is denoted by $f_D(r_t|s_t = k, \Psi, \mathcal{F}_{t-1})$. In this study the standardized innovations, $\eta k, t$, of the model in each regime is assumed to be conditional to three distributions, that is, the normal, generalized error distribution(GED) and student-t distributions. The standardization is such that each distribution has zero mean and a unit variance. For notational purposes,the time and regime indices are dropped but the shape parameters are conditional on the regimes. The maximum likelihood estimator $\tilde{\Psi}$ is thus obtained by maximizing equation (2.6).

Normal distribution. The standardized normal distribution has the density function given by

$$f_N(\eta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\eta^2}. \tag{2.8}$$

Student-t distribution. Let x_v be a student-t distribution with v degrees of freedom, then $var(x_v) = v/(v-2) for v > 2$. If we let $\eta = x_v/\sqrt{v/(v-2)}$, then the probability density function of η is

$$f_S(\eta|v) = \frac{\Gamma(v+1)/2}{\Gamma(v/2)\Gamma(v-2)\pi}\Big[1+\frac{\eta^2}{v-2}\Big]^{-(1+v)/2}, v > 2 \text{where} \Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx. \tag{2.9}$$

It is required that $v > 2$ so that the second moment exists.

Generalized error distribution(GED). The standardized generalized error distribution (GED) has the density function given by

$$f_G ED(\eta; v) = \frac{ve^{-\frac{1}{2}|\frac{\eta}{\lambda}|^v}}{\lambda 2^{(1+1/v)}\Gamma(1/v)},\ 1 < \eta < \infty,\ \text{where}\ \lambda = \Big[\frac{\Gamma(1/v)}{4^{(1/v)}\Gamma(3/v)}\Big]^{1/2} \tag{2.10}$$

where $v > 0$ is the shape parameter. The GED yields the normal distribution if $v = 2$,and if $v < 2$, the density function has thicker tails than the normal density function, whereas for $v > 2$, it has thinner tails.

3. Results and discussion

3.1. **Empirical data.** The data set utilized by this study comprises the daily and weekly closing indices, and the corresponding trading volume as reported in S&P500 index(developed market) and the Nairobi Securities exchange index (developing market). The daily and weekly log returns $\{R_t\}$ and the corresponding trading volume $\{V_t\}$ at time t are computed for the period ranging from January 2, 2001 to December 29, 2017 for the two indices.

3.2. **Descriptive statistics.** Table 1 is a presentation of the basic statistical properties of the index returns and trading volume which reports that all the index return series have a positive mean which is close to zero and this is an indication of a realization of positive returns on the investment. Further, we find that the mean and variance slightly increases as the return series change from daily to weekly. The returns series from S&P500 stock market index are all negatively skewed as compared to returns from the developing stock market (NSE20 index) which has positive skewness. This is an indication that the distribution of S&P500 index returns has heavy left tail than the right tail whereas that of NSE20 index return has a heavy right tail as compared to its left tail. The basic statistics for trading volume indicate that the mean of both frequencies is positive except for the weekly S&P500 volume in addition to being close to zero. Moreover,the trading volume has negative skewness except for the weekly S&P500 volume frequency.

Table 1. Basic statistics

Index		Obs	Mean	Std dev	Skew	Ex.Kurt	JB
S&P500	Returns	Daily	0.00017	0.01202	-0.21989	6.38927	15,757
		Weekly	0.00084	0.02379	-0.92811	5.09105	2,651
	Volume	Daily	0.00420	0.16002	-0.03913	-2.5479	14,508
		Weekly	-0.00108	0.25109	0.09257	4.79695	2,341
NSE20	Returns	Daily	0.00016	0.00857	0.39295	8.32209	22,949
		Weekly	0.00076	0.02569	0.41755	2.87156	1,297
	Volume	Daily	0.01033	1.03601	-0.23194	-1.71409	888
		Weekly	0.00392	0.88029	-0.00651	-1.39927	95

A further description of the data is carried out by plotting the empirical density versus the normal distribution, and quantile-quantile (qq) plot of the data as depicted in figures 1, 2, 6 and 7. Clearly, the plots indicate that the empirical density does not come from a normal distribution hence the data does not follow normal distribution. The non-normality of the returns and volume is further supported by the Jarque-Bera(JB) statistic in table 1.

A test for stationarity and ARCH effects is investigated by using the Augmented Dickey Fuller (ADF) and ARCH-LM tests respectively. Performing these tests is a necessary undertaking before applying GARCH model to the data. The ADF and ARCH-LM tests results from table 3 clearly rejects the null hypotheses of a unit root and no ARCH effects respectively. That is, the return series are stationary and there are ARCH effects.

3.3. **Empirical findings and discussion.** The time series plots of the stock prices, trading volume and index returns are shown in figures 3 to 5. Evidently, the plots display the common stylized facts of time series data set such as clustering

Table 2. ADF and ARCH-LM tests for indices returns

		Test	Statistic	P-value	Lag Order
Daily returns	S&P500	ADF-Test	-17.67	0.01	12
		ARCH LM-Test	1299.8	<2.2e-16	12
	NSE20	ADF-Test	-16.51	0.01	12
		ARCH LM-Test	1028	<2.2e-16	12
Weekly returns	S&P500	ADF-Test	-8.766	0.01	12
		ARCH LM-Test	151.1	<2.2e-16	12
	NSE20	ADF-Test	-6.300	0.01	12
		ARCH LM-Test	86.66	2.184e-13	12

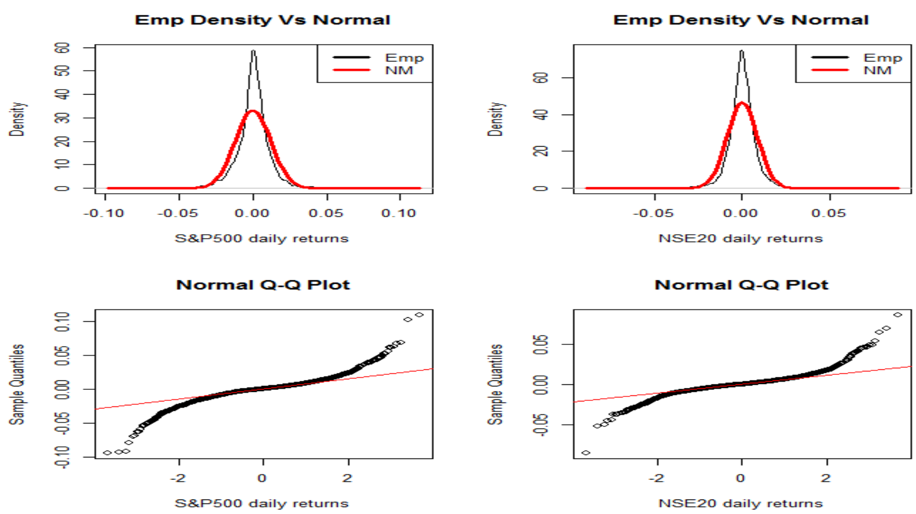

Figure 1. Empirical density versus normal distribution, and qq-plots for daily returns

of volatility and presence of outliers. Clearly, the implication is that the mean and variance of the data are changing hence it can be concluded that the time series data is not stationary. Furthermore, large and small returns are changing alternatively across the time interval of the sample which can be construed to mean that conditional variance is changing with time in accordance with a regime switching specification.

The parameter estimates of GARCH(1,1), presented in tables 3 and 6(in the appendix), satisfy the conditions that $\omega_1 > 0$, $\alpha_1 > 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$ which is necessary for mean reverting process. This implies that the model is weakly stationary and that all the conditional volatilities are mean reverting for the time series. The model coefficients, ω, α_1 and β_1, are highly significant for all the indices returns. Moreover, the value of β_1 is large in both daily and weekly indices return series of the developed market than in the daily and weekly indices return series of the emerging market. This implies that large values of σ^2_{t-1} are followed by large values of σ^2_t, and small values of σ^2_{t-1} are followed by small values of σ^2_t. That is, the

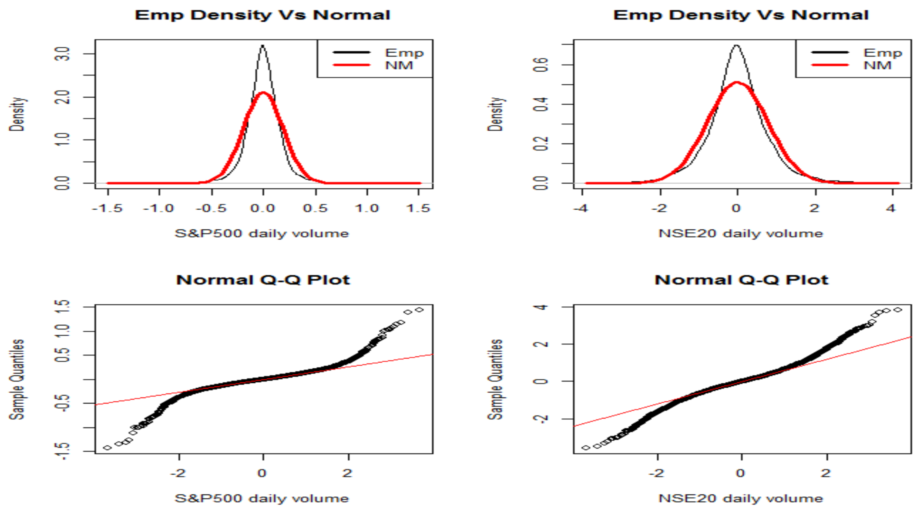

FIGURE 2. Empirical density versus normal distribution, and qq-plots for daily volume

developed market (S&P500) is characterized by high volatility clustering compared to low volatility clustering in the developing market (NSE20). It also means that shocks to conditional variance takes long to die off in S&P500 index return series as compared to the NSE20 index returns. This is further confirmed by a look at the sum of α_1 and β_1 which is close to 1 and high in the S&P500 indices return series as compared to the NSE20 indices returns series. The value of $\alpha_1 + \beta_1$ being close to 1 is an implication that volatility is highly persistent in S&P500 index than in NSE20 index. Again, it is noted that as the data changes from daily to weekly, the volatility persistence decreases in both indices returns.

The parameter estimates of MSGARCH (1,1) are presented in tables 4 and 7(in the appendix). The parameters are different across regimes and most of them are significant at 1% significance level. For the S&P500 daily returns, regime 1 reports low values of $\alpha_{1,1} + \beta_1$ compared to $\alpha_{1,2} + \beta_2$ though the values are very close across the regimes. This is to mean, volatility persistence in regime 1 is low than in regime 2. On the other hand, NSE20 index returns reports regime 1 to have high volatility persistence than regime 2, though the parameter estimates for GED distribution reports otherwise. Considering the results for weekly returns, S&P500 index returns reports regime 1 to have higher volatility persistence compared to regime 2, whereas NSE20 index returns reports the opposite case for S&P500. In general, similar behavior of volatility persistence in both regimes can be inferred between the daily S&P500 and weekly NSE20 indices returns, and between the weekly S&P500 and daily NSE20 indices returns. Furthermore, the results show that the volatility process evolution across the regimes is heterogeneous in addition to existence of different unconditional volatility levels. A look at the transition probabilities reports that the volatility process has the tendency to spend more time in first regime than in regime 2. That is, the probability of the volatility process to stay in regime 1 before transiting to second regime is high compared to the probability of reverting back once in regime 2.

Tables 5 and 8 reports the parameter estimates of GARCH (1,1) with lagged trading volume added in the conditional variance equation as an exogenous variable. The estimated parameters are in general statistically significant at 1% significance level. The results for volatility persistence, that is, $\alpha_1 + \beta_1$ are compared with similar results as reported by GARCH (1,1) without an exogenous variable(see tables 3 and 6) and a general inference is made that, volatility persistence decreases in both S&P500 and NSE20 indices returns except that the persistence increases for the NSE20 weekly index returns. In GARCH(1,1), the trading volume coefficient,δ, is positive, a suggestion that trading volume explains volatility whereas the increase in persistence of volatility after including trading volume is an indication that asymmetric volatility on the market is a consequence of trading volume.

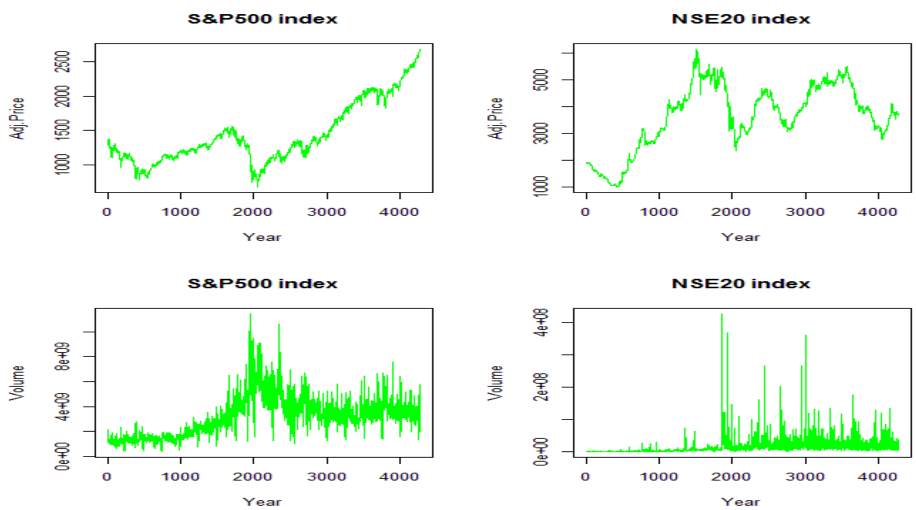

Figure 3. Daily S&P500 and NSE20 Adj closing prices & Volume

Table 3. GARCH(1,1) estimates for daily indices returns

	S&P500			NSE20		
Estimate	Nm	St	Gd	Nm	St	Gd
μ_1	0.0005***	0.0007***	0.0006***	0.0001	0.0000	0.0000
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
α_1	0.0976***	0.0996***	0.0991***	0.2675***	0.3642***	0.3157***
β_1	0.8883***	0.8989***	0.8936***	0.6920***	0.5232***	0.5986***
$\alpha_1 + \beta_1$	0.9858	0.9985	0.9927	0.9595	0.8874	0.9243

[1]Note: Nm, St & Gd refers to the normal, students-t and generalized error distributions respectively, whereas the asterisks *, ** and *** stand for 10%, 5% and 1% α-level of significance respectively.

4. Conclusion

This study investigated the dynamic correlation between stock index returns, volatility and trading volume in the developed and developing financial market by

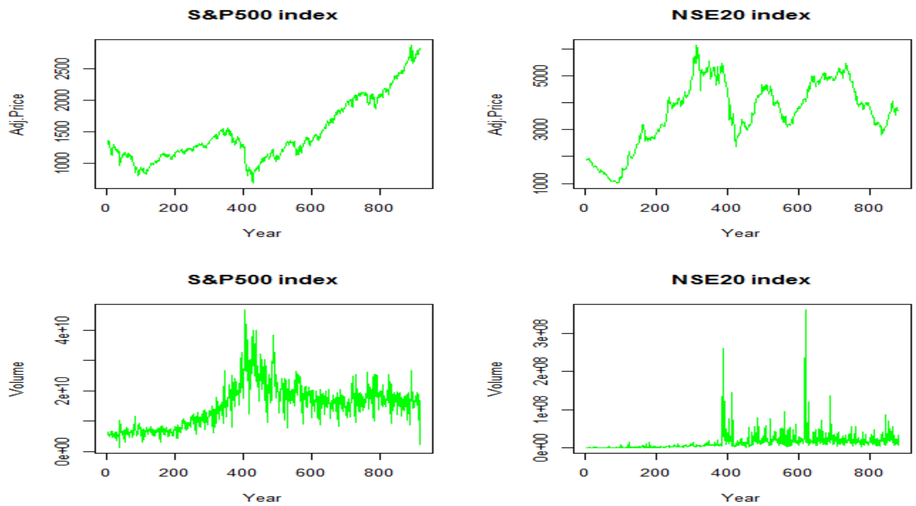

FIGURE 4. Weekly S&P500 and NSE20 Adj closing prices & Volume

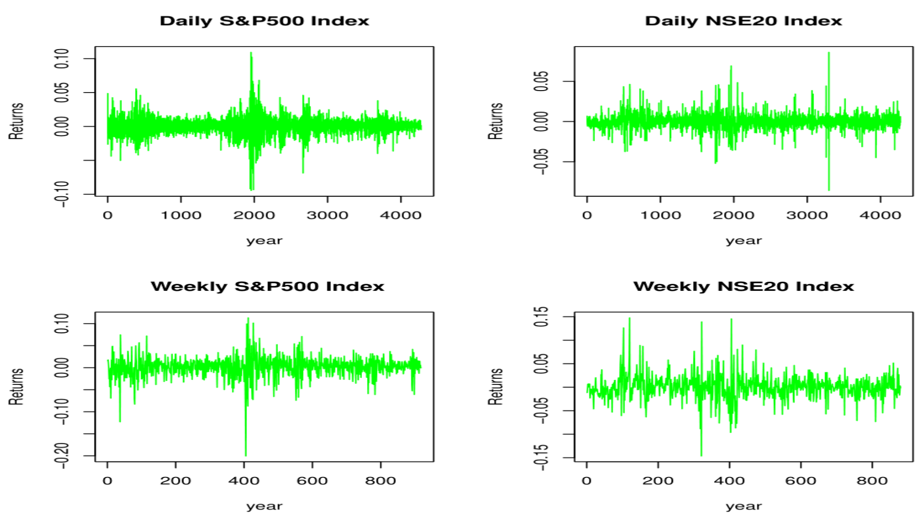

FIGURE 5. Daily & weekly S&P500 and NSE20 indices returns

applying GARCH and Markov-switching GARCH models. Moreover, the effect of adding trading volume in the conditional variance equation of GARCH model is examined. The results shows that the stock returns and volume have common characteristics of financial time series. The data exhibits a considerable kurtosis which we relate to the time-dependence in conditional variance and also the distribution of all the series is relatively asymmetric. All the series exhibit asymmetric behavior in the conditional variance which can be attributed to leverage effects. The results further reveal existence of strong persistence of volatility and that the previous volatility can explain the present volatility. However, the volatility persistence reduces as

TABLE 4. MSGARCH(1,1) estimates for daily indices returns

	S&P500			NSE20		
Estimate	Nm	St	Gd	Nm	St	Gd
$\alpha_{0,1}$	0.0000***	0.0000	0.0000***	0.0000***	0.0000***	0.0000***
$\alpha_{1,1}$	0.0362***	0.1107	0.0283***	0.0071	0.0151	0.2737***
β_1	0.9499***	0.7819***	0.9705***	0.9463***	0.9301***	0.6285***
nu_1		3.123***	1.363***		8.874***	1.690***
$\alpha_{0,2}$	0.0000***	0.0000***	0.0000***	0.0001***	0.0000***	0.0001***
$\alpha_{1,2}$	0.1258***	0.0927***	0.0780***	0.2932***	0.5480***	0.9999***
β_2	0.8699***	0.8954***	0.8900***	0.3565***	0.2862***	0.0000***
nu_2		9.771***	1.784***		5.771***	0.7000***
P_{11}	0.9389***	0.9959***	0.9918***	0.9624***	0.9672***	0.9695***
P_{21}	0.2074***	0.0009	0.0220***	0.1261***	0.0392***	0.1423***
$\alpha_{1,1}+\beta_1$	0.9861	0.8926	0.9988	0.9534	0.9452	0.9022
$\alpha_{1,2}+\beta_2$	0.9957	0.9881	0.9680	0.6497	0.8342	0.9999

[2]Note: Nm, St & Gd refers to the normal, students-t and generalized error distributions respectively, whereas the asterisks *, ** and *** stand for 10%, 5% and 1% α-level of significance respectively.

TABLE 5. GARCH(1,1) for daily indices returns with trading volume

	S&P500			NSE20		
Estimate	Nm	St	Gd	Nm	St	Gd
μ_1	0.0005***	0.0006***	0.0006***	0.0001***	0.0000	0.0000
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
α_1	0.0963***	0.1102***	0.0960***	0.2639***	0.3535***	0.3104***
β_1	0.8886***	0.8794***	0.8940***	0.6923***	0.5355***	0.6019***
δ	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***
$\alpha_1+\beta_1$	0.9849	0.9896	0.9900	0.9562	0.8890	0.9123

[3]Note: Nm, St & Gd refers to the normal, students-t and generalized error distributions respectively, whereas the asterisks *, ** and *** stand for 10%, 5% and 1% α-level of significance respectively.

the data changes frequency from daily to weekly. Furthermore, the study reports some stylized facts of the data such as volatility clustering, leptokurtic distribution and leverage effects. The MSGARCH (1,1) results reports a heterogeneous volatility process evolution across the two regimes for the two indices returns. Indeed both regimes are characterized by heterogeneous unconditional volatility, volatility persistence as well as different reaction to negative returns. Moreover, most of the transition probabilities report that the volatility process has the tendency to spend more time in regime 1 before transiting to the next state. That is, the rate of the process to change from regime 1 to regime 2 is slow compared to the rate of reverting back once in the second regime.

The study further reveals that inclusion of lagged trading volume in the conditional variance equation of GARCH (1,1) model lead to a decrease in volatility persistence for the two market indices except in the weekly NSE20 index. The increase in persistence of volatility after including lagged trading volume in the conditional variance equation is an indication that asymmetric volatility on the market is a consequence of trading volume. On the other hand, in GARCH (1,1) model, the trading volume coefficient, δ, is positive, and this suggests that trading volume explains volatility.

Appendix

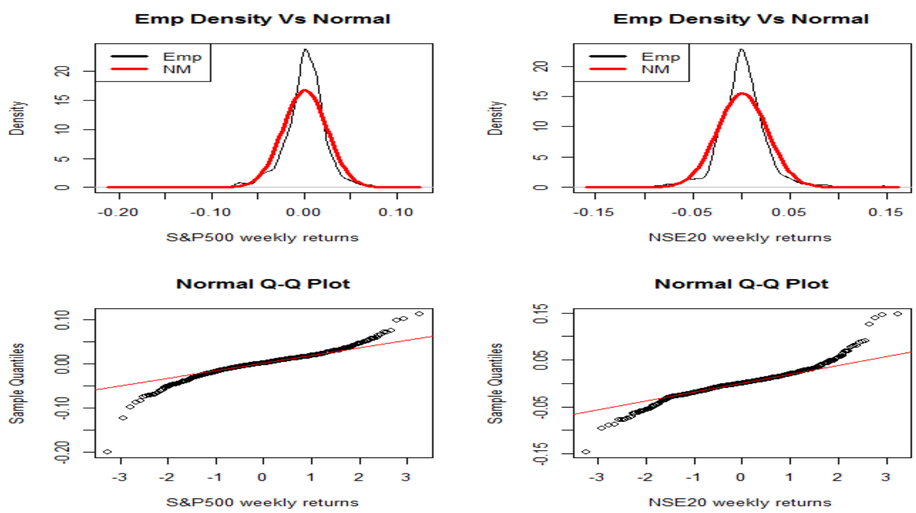

Figure 6. Empirical density versus normal distribution, and qq-plots for daily returns

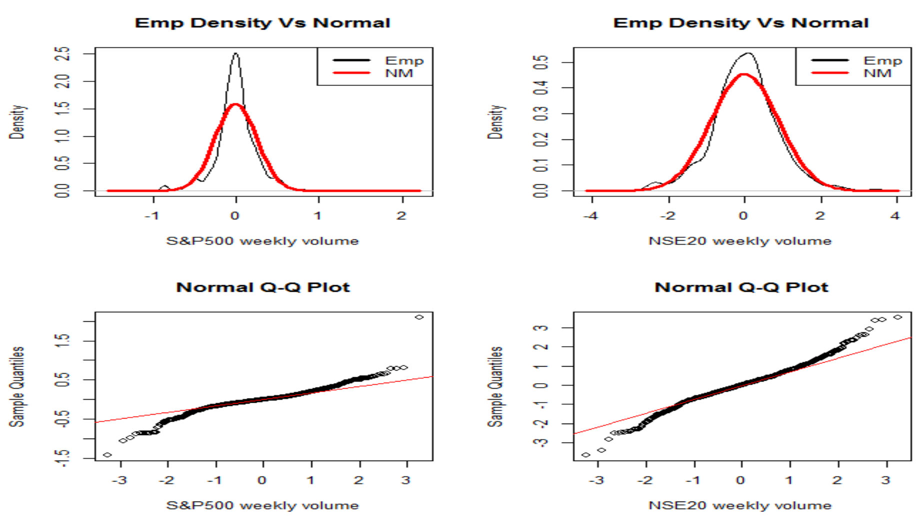

Figure 7. Empirical density versus normal distribution, and qq-plots for daily volume

Table 6. GARCH(1,1) estimates for weekly indices returns

	S&P500			NSE20		
Estimate	Nm	St	Gd	Nm	St	Gd
μ_1	0.0024***	0.0026***	0.0024***	0.0004	0.0009	0.0005
ω	0.2781***	0.0000**	0.0000**	0.0001***	0.0001***	0.0001***
α_1	0.2240***	0.1572***	0.1891***	0.3435***	0.4196***	0.3764***
β_1	0.7379***	0.8137***	0.7744***	0.4833***	0.4330***	0.4513***
$\alpha_1+\beta_1$	0.9619	0.9709	0.9635	0.8268	0.8526	0.8277

[4]Note: Nm, St & Gd refers to the normal, students-t and generalized error distributions respectively, whereas the asterisks *, ** and *** stand for 10%, 5% and 1% α-level of significance respectively.

Table 7. MSGARCH(1,1) estimates for weekly indices returns

	S&P500			NSE20		
Estimate	Nm	St	Gd	Nm	St	Gd
$\alpha_{0,1}$	0.0130***	0.0130***	0.0177***	0.0001**	0.0000	0.0001***
$\alpha_{1,1}$	0.3015***	0.3028***	0.2893***	0.0914	0.0474	0.4939***
β_1	0.0002	0.0002	0.0002	0.6071***	0.6703***	0.3298***
nu_1		99.81***	1.4831***		99.50***	1.986***
$\alpha_{0,2}$	0.1142	0.1162	0.1760***	0.0002	0.0001	0.0000
$\alpha_{1,2}$	0.0001	0.0000	0.0000	0.1914	0.1117	0.0308
β_2	0.2318	0.2123	0.0073***	0.7955***	0.8731***	0.9580***
nu_2		99.53*	2.173***		10.78***	0.9289***
P_{11}	0.8244***	0.8248***	0.8964***	0.8761***	0.8777***	0.6993***
P_{21}	0.4930***	0.4870***	0.4224***	0.3422***	0.2008***	0.4545***
$\alpha_{1,1}+\beta_1$	0.3017	0.3030	0.2895	0.6985	0.7177	0.8237
$\alpha_{1,2}+\beta_2$	0.2319	0.2123	0.0073	0.9869	0.9848	0.9888

[5]Note: Nm, St & Gd refers to the normal, students-t and generalized error distributions respectively, whereas the asterisks *, ** and *** stand for 10%, 5% and 1% α-level of significance respectively.

Table 8. GARCH(1,1) for weekly indices returns with trading volume

	S&P500			NSE20		
Estimate	Nm	St	Gd	Nm	St	Gd
μ_1	0.0021***	0.0023***	0.0023***	0.0005	0.0009	0.0005
ω	0.0000***	0.0000***	0.0000***	0.0000***	0.0001***	0.0001***
α_1	0.1815***	0.1246***	0.1533***	0.2432***	0.3545***	0.2960***
β_1	0.7597***	0.8344***	0.7962***	0.6168***	0.5005***	0.5443***
δ	0.0002***	0.0002***	0.0002***	0.0001***	0.0001**	0.0001***
$\alpha_1+\beta_1$	0.9412	0.9590	0.9495	0.8600	0.855	0.8403

[6]Note: Nm, St & Gd refers to the normal, students-t and generalized error distributions respectively, whereas the asterisks *, ** and *** stand for 10%, 5% and 1% α-level of significance respectively.

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