

## QUANTIFYING ECONOMIC RISK: AN APPLICATION OF EXTREME VALUE THEORY FOR MEASURING FIRE OUTBREAKS FINANCIAL LOSS.

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ABSTRACT. Using financial losses data as a result of fire breakout, this paper models the distribution of the events loss probability and estimates the various quantiles and the tail distribution of the available data. The paper also models the extreme through maximum threshold to obtain useful measurements of the Value at Risk (VaR) and the Expected Short-Fall (ES) at 90%, 95%, 98% and 99%. Notably, the study shows, the Value at Risk at 99% is GHS 30,239,067 with Expected Shortfall of GHS 28,891,466. The return level for 5 years and 50 years were all found to be in excess of GHS 30 million.

### 1. INTRODUCTION

Extreme value theory is a known statistical theory which suites the unusual behavior such as rare natural disasters. The theory is the bedrock of scientist quest to allocate resources planning in everyday economics activities which are rare, and to predict the likelihood that an exceptional extreme event might occur in not too distant future. Extreme value models use an asymptotic approximation for tail distributions from the Pareto probability density function, models emanating from this theory have shown accuracy in results forecasting unusual events in finance. Primarily, the objective of Extreme Value theory is to assess, from a series of observations, the probability of events that are more extreme than those previously recorded, with the advent of new technology, predictive analytics has gone in a long way by supporting the predictive nature of extreme value theory.

The foundation for Extreme Value Theory (EVT) was laid by Fisher and Tippett (1928) modelling and quantifying phenomena where events were rare resulting in scarcity of data or unavailability of data in itself, after nearly two decades, Gnedenko (1943) formalized the previous work of Fisher and Tippet and helped form what is now known as the extreme value condition. Nevertheless, Gumbel (1958) became the first person to applied this innovative statistical modeling technique to

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2000 *Mathematics Subject Classification.* 35A07, 35Q53.

*Key words and phrases.* Extreme value theory; Quantifying economic risk; Financial loss; Fire outbreaks.

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Submitted April 26, 2019. Published September 11,2019.

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Communicated by Fazlollah Soleymani.

estimate extremes and hence the resultant of the Gumbel distribution which was named after him.

The work by Beirlant et al. (2004) throw light on the theoretical aspects and credited de Haan (1970) for finding the link of the properties of sample extremes to the central limit theorem for the sample mean. These advances lead to the growing interest in the field and had led to various aspects of the theorem being developed. Such practical events are seen in many fields which includes insurance (Embrechts et al. 1997), finance (Embrechts et al. 1997; Gilli and Killezi 2006), environmental science (Eastoe and Tawn 2009; Katz 2010), sport science (Einmahl and Magnus 2008; Henriques-Rodrigues et al. 2011), metallurgy (Beirlant et al. 2004), earth sciences (Dargahi-Noubary 1986; Pisarenko and Sornette 2003), engineering (Castillo et al., 2004) and environmental science (Reiss and Thomas, 2007).

## 2. FIRE OUTBREAKS AND ECONOMIC ACTIVITIES

Fire outbreak in unstructured market infrastructure is common in developing countries, It is a common knowledge that Small and Medium Enterprises file for bankruptcy once engulf by fire, these fold up of companies does not depends on bankruptcy models or solvency ratios, hence the mitigation process available to remain in business after this extreme event is through insurance.

Insurance claims data is known to have a thicker tails, therefore insurers try to identify potential risk of huge claims such as fire outbreak. In Ghana such events are unpredictable, hence modeling of such scenarios by predicting with some level of coincidence helps managing huge loses to keep business in place and contribute to the thriving of national economy.

Though the complexity of economic impacts of disasters make it difficult to evaluate (Okuyama, 2008), the destruction of properties release an untold economic suffering on various households, however, scholars agree that, such disasters exerts significant pressure on budgetary allocations at macroeconomic level and slows community business with both narrowly fiscal short term impacts and wider long term development implications. This has been attributed to lack of alternative resources to response to disasters and falling on reallocation of primary fiscal response disaster (Benson & Clay, 2003; Okuyama, 2008). (Pelling & Wisner, 2009) argues that, count of loses after disasters are related to direct losses alone, and does not account for indirect losses leading to inferior effects that continuously affects disaster victims through the period of recovery. These leads to reduced employment opportunities and contributes to knock-on indirect effects through reduction in investment, reduced productivity capacity, reduced consumption and incurred cost of resettlement and most importantly introduce dead weight losses to SMEs affected. These effects could be eliminated or minimized through planning by insurance firms for such events and makes provision to revive the economies of the affected areas. In measuring the financial risk within a company over a period of time, Value at Risk (VaR) is mostly employed to estimate the extent of losses should such an event occurred, the study seek to estimate the Value at Risk, Expected Shortfall and Return Levels to fire outbreaks financial loss data in order to prepare insurance companies for such high severity events thereby allocating cash reserves to cover these losses or through re-insurance.

## 3. EXTREME VALUE THEORY (EVT)

Let  $X_1, X_2, \dots, X_N$  be a random sequence of independently and identically distributed variables with distribution function  $F$ . Let the ordered statistics for the sequence also be defined as  $X_{1,N} \leq X_{2,N} \leq \dots \leq X_{N,N}$ . If the object of interest is either minimum or maximum from the ordered sequence, then the maximum could be defined as

$$X_{N,N} = \max\{X_1, X_2, \dots, X_N\} \quad (3.1)$$

and the minimum is also defined as

$$X_{1,N} = \min\{X_1, X_2, \dots, X_N\} = -\max\{-X_1, -X_2, \dots, -X_N\} \quad (3.2)$$

Therefore, the distribution function of the maximum  $X_{N,N}$  or minimum  $X_{1,N}$  is related to the distribution function of the sequence  $F$  shown as:

$$F_{1,N}(x) = F^N(x) \quad (3.3)$$

The nature of  $F$  is unknown and therefore, from EVT,  $F^N(x)$  could be approximated with limit distribution as  $n \rightarrow \infty$ .

**Theorem 3.1.** *If there exist a sequence  $a_n > 0$  and  $b_n \in R$  such that (Fisher-Tippet, 1928 Theorem)*

$$\lim_{n \rightarrow \infty} P\left(\frac{X_{N,N} - b_n}{a_n}\right) \rightarrow G_y(x) \quad (3.4)$$

Where  $G$  is a non-degenerate function, the  $G$  belongs to one of the extreme value distribution given as:

$$[I] G_y(x) = \exp\left(-\exp\left(-\frac{x-b}{a}\right)\right), x \in R (\gamma = \alpha = 0) \quad (3.5)$$

$$[II] G_y(x) = \begin{cases} 0, & \text{if } x \leq b \\ \exp\left(-\left(\frac{x-b}{a}\right)^{-\alpha}\right), & \text{if } x > b, \alpha > 0 (\gamma = \frac{1}{\alpha} > 0) \end{cases} \quad (3.6)$$

$$[III] G_y(x) = \begin{cases} 1, & \text{if } x \geq b \\ \exp\left(-\left(-\frac{x-b}{a}\right)^\alpha\right), & \text{if } x < b, \alpha > 0 (\gamma = -\frac{1}{\alpha} < 0) \end{cases} \quad (3.7)$$

For all  $a > 0$  and  $b \in R$

These limiting distributions are known as Gumbel, Pareto and Weibull extreme value distributions respectively. The Generalized of the above distributions is referred to us Generalized Extreme Value (GEV) distribution with the probability density function shown as:

$$G_y(x) = \begin{cases} \exp\left(-\left(1 + \gamma\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{1}{\gamma}}\right), & 1 + \gamma\left(\frac{x-\mu}{\sigma}\right) > 0, \gamma \neq 0 \\ \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right), & x \in R, \gamma = 0 \end{cases} \quad (3.8)$$

Where  $\mu$ ,  $\sigma$  and  $\gamma$  are the location, scale and shape parameters or the tail index. The probability density function derived from the distribution function of equation 3.8 is expressed as;

$$f(x) = \begin{cases} \frac{1}{\sigma} \left(1 + \gamma \left(\frac{x - \mu}{\sigma}\right)\right)^{\frac{-1+\gamma}{\gamma}} * \exp \left\{ - \left[1 + \gamma \left(\frac{x - \mu}{\sigma}\right)\right]^{\frac{-1}{\gamma}} \right\} & \gamma \neq 0 \\ \frac{1}{\sigma} \exp \left( - \left[ -\frac{x - \mu}{\sigma} + \exp \left( -\frac{x - \mu}{\sigma} \right) \right] \right) & \end{cases} \quad (3.9)$$

Special distribution are derived from the density functions,  $\gamma = 0, \gamma > 0$  and  $\gamma < 0$  corresponds to the Gumbel distribution including exponential tailed distributions, normal, gamma and log-normal distributions, fat tailed distributions such as Cauchy and Pareto and the short tailed distributions which includes the uniform, beta and Weibull domains of attractions respectively.

### 3.1. Parameter Estimation.

Over the past two decades, different methods have been developed to estimate the parameters of GEV such as the L-moment methods (Hosking, 1990), the method of moments (Christopher, 1994), the L-method of moments with less influence by outliers (Hosking and Wallis, 1997), robust estimates L-moment methods (Von Storch and Zwiers, 1999), and as well as the Bayesian method (Smith and Naylor, 1987, Lye et al. 1993, Coles and Tawn, 2005), however, the most popular among all these is the maximum likelihood method (Smith and Naylor, 1987; Unkašerić and Tošić, 2009) which is applied to this study.

Let  $X_1, X_2, \dots, X_N$  be a maxima random variable with GEV distribution with the probability density function defined in equation 3.9 with  $\gamma \neq 0$ , the likelihood estimation method follows as:

$$L(\gamma, \sigma, \mu; X) = \prod_{i=1}^n \left[ \frac{1}{\sigma} \left(1 + \gamma \left(\frac{x - \mu}{\sigma}\right)\right)^{\frac{-1+\gamma}{\gamma}} * \exp \left\{ - \left[1 + \gamma \left(\frac{x - \mu}{\sigma}\right)\right]^{\frac{-1}{\gamma}} \right\} \right] \quad (3.10)$$

$$= \frac{1}{\sigma^n} \prod_{i=1}^n \left[ \left(1 + \gamma \left(\frac{x - \mu}{\sigma}\right)\right)^{\frac{-1+\gamma}{\gamma}} * \exp \left\{ - \left[1 + \gamma \left(\frac{x - \mu}{\sigma}\right)\right]^{\frac{-1}{\gamma}} \right\} \right] \quad (3.11)$$

Taking the logarithm of the likelihood function leads to

$$l(\gamma, \sigma, \mu; X) = n \log \sigma - \left(1 + \frac{1}{\gamma}\right) \sum_{i=1}^n \log \left(1 + \gamma \left(\frac{x_i - \mu}{\sigma}\right)\right) - \sum_{i=1}^n \left[1 + \gamma \left(\frac{x_i - \mu}{\sigma}\right)\right]^{\frac{-1}{\gamma}} \quad (3.12)$$

The maximization of the function is realized under the constraint  $1 + \gamma \left(\frac{x_i - \mu}{\sigma}\right) > 0$  and  $\sigma > 0$ . The properties of consistency, asymptotic efficiency and normality holds as explained by smith (1989) as n gets large. For  $\gamma = 0$ , the logarithm of the likelihood function is given by

$$l(\gamma, \sigma, \mu; X) = n \log \sigma - \sum_{i=1}^n \exp \left( -\frac{x_i - \mu}{\sigma} \right) - \sum_{i=1}^n \left( \frac{x_i - \mu}{\sigma} \right) \quad (3.13)$$

And thereby applying the partial derivatives with respect to the two parameters leads to the systems of equation

$$\begin{cases} n - \sum_{i=1}^n \exp\left(-\frac{x_i - \mu}{\sigma}\right) = 0 \\ n + \sum_{i=1}^n \left(\frac{x_i - \mu}{\sigma}\right) \left[\exp\left(-\frac{x_i - \mu}{\sigma}\right) - 1\right] = 0 \end{cases} \quad (3.14)$$

However, its worth noting that, no closed form solution exist and this can be solve by numerical methods.

### 3.2. Return Period for Generalized Extreme Value.

The determination of the mean waiting time between extreme events is defined by the equation of the return level. The T year return is given as the solution of the equation

$$\begin{aligned} F_x(z_T) = P(X \leq z_T) &= 1 - \frac{1}{T} \\ \implies z_T &= F^{-1}\left(1 - \frac{1}{T}\right) \end{aligned} \quad (3.15)$$

Where  $z_T$  is the return period or the level of exceedance by the annual maximum in every  $T$  years on average. For any given  $T$  years the following quantiles are derived from equation 3.8 as shown below;

$$z_T = \begin{cases} \mu + \frac{\sigma}{\gamma} \left\{1 - \log\left(1 - \frac{1}{T}\right)\right\}^\gamma, \gamma \neq 0 \\ \mu - \sigma \log\left\{-\log\left(1 - \frac{1}{T}\right)\right\}, \gamma = 0 \end{cases} \quad (3.16)$$

### 3.3. The Generalized Pareto Distribution (GPD).

To estimate peaks over threshold method, Hoskings and Wallis (1987) proposed the two parameter distribution called the generalized the Pareto distribution. The function is given as

$$G_{\gamma, \sigma} = \begin{cases} 1 - \left(1 + \frac{\gamma x}{\sigma}\right)^{\frac{-1}{\gamma}}, \gamma \neq 0 \\ 1 - \exp\left(-\frac{x}{\sigma}\right), \gamma = 0 \end{cases} \quad (3.17)$$

Where  $\sigma > 0, x \geq 0$  and  $0 \leq x \leq \frac{\sigma}{\gamma}$  and  $\gamma < 0$  Based on the equation, the excess distribution could be obtained using McNeil theorem (McNeil, 1999).

**Theorem 3.2.** *Let  $\mu$  be a large enough threshold, the distribution function of excess losses over  $\mu, (X - \mu)$  provided  $X > \mu$  is defined as*

$$F_u(y) = P(X - \mu \leq | X > \mu) \quad (3.18)$$

For  $0 \leq y \leq x_0 - \mu$ . By estimating the loss distribution using maximum likelihood method of the GPD arrives at,

$$\hat{F} = 1 - \frac{N_\mu}{n} \left(1 + \gamma \frac{x - \mu}{\sigma}\right)^{\frac{-1}{\gamma}} = P(X < x | X > \mu) \quad (3.19)$$

Where  $N_\mu$  the number of exceedance of the threshold is,  $\mu$  is the threshold and  $n$  is the total observation of the data. The resulting probability density function is derived as

$$\hat{f} = \frac{N_\mu}{n\sigma} \left(1 + \gamma \frac{x - \mu}{\sigma}\right)^{-1 - \frac{1}{\gamma}} = P(X < x | X > \mu) \quad (3.20)$$

#### 3.4. Maximum Likelihood Estimation for GPD with exceedance.

Let  $X_1, X_2, \dots, X_n$  be a sequence of independently and identically distributed random variables. If  $N_\mu$  is the number of observations above the threshold  $\mu$ , then the likelihood function is given as

$$\log L(\sigma_\mu, \gamma | X) = -N_\mu \log \sigma_\mu - \left(1 + \frac{1}{\gamma}\right) \sum_{i=1}^{N_\mu} \log \left[1 + \gamma \frac{x_i - \mu}{\sigma_\mu}\right] \quad (3.21)$$

Where  $\left[1 + \gamma \frac{x_i - \mu}{\sigma_\mu}\right] > 0$  and  $\gamma \neq 0$ , where  $\gamma = 0$ , the likelihood function can be derived to obtained

$$\log L(\sigma_\mu | X) = -N_\mu \log \sigma_\mu - \sum_{i=1}^{N_\mu} \log \left(\frac{x_i - \mu}{\sigma_\mu}\right) \quad (3.22)$$

#### 3.5. Estimation of Returns Levels from the GPD, Value at Risk (VaR) and Expected Shortfall.

Given the shape and scale parameters is of GPD model with a suitable threshold with exceedances, then a random variable  $X$  over a high threshold is given as:

$$P(X < x | X > \mu) = \left(1 + \gamma \frac{x_i - \mu}{\sigma_\mu}\right)^{\frac{-1}{\gamma}} \quad (3.23)$$

And the corresponding value at risk estimated as

$$VaR_q = F^{-1}(q) = \mu + \frac{\sigma}{\gamma} \left( \left( \frac{n}{N_\mu} (1 - q) \right)^{-\gamma} - 1 \right) \quad (3.24)$$

Where  $q$  the quantile and the expected short fall is estimated as

$$ES_q = E(X | X \geq VaR_q) \quad (3.25)$$

$$ES_q = VaR_q + E(X - VaR_q | X \geq VaR_q)$$

Through mathematical substitution will lead to

$$\frac{ES_q}{VaR_q} = \frac{1}{1 - \gamma} + \frac{\sigma - \gamma\mu}{(1 - \gamma)VaR_q} \quad (3.26)$$

$$ES_q = \frac{VaR_q}{1 - \gamma} + \frac{\sigma - \gamma\mu}{(1 - \gamma)} \quad (3.27)$$

4. RESULTS AND DISCUSSION

4.1. Summary Statistics and Data Distribution Fitting.

Data from financial loss of properties due to fire outbreak were collected from various government offices mandated to hold data for such occurrences from 2001 to 2016 in Ghana. These data were not holistic nature as most properties destroyed during fire outbreaks were not insured hence claims cannot be made, in these cases, the estimated cost for the loss of properties were used from the Ghana fire service department. Data were gathered from all the regional and national centers of both the National Disaster Management Organization and the Ghana Fire Service, The data contains 167 event data points for various financial losses during fire outbreaks in Ghana.

The summary statistics data illustrated in Table 1 shows a highly positively skewed and very peaked data with high variability spread of the financial losses within the study period. The average was found to be GHS 596,280 over the period with the median found to be GHS 20,104. In studies under this nature, it is better to use the median rather than the mean value, as mean values tend to be bias and less robust as compared to the median value due to the influences of extremities. The percentile of the data shows a much fat tail distribution as the 95th percentile is in excess of  $4.049 \times 10^6$ .

TABLE 1. Summary Statistics for Financial Loss Data for Fire Outbreaks

Min	1st Quant	Median	Mean	3rd Quant	Variance
1728	7299	20104	596280	47675	$1.06 \times 10^{14}$
Max	5th Percentile	10th Percentile		90th Percentile	95th Percentile
7523654	1972.7260	2190.180		3067348.20	4049905.9

Estimating the parameters with maximum likelihood method, the location parameter, shape parameter and scale parameter for the Weibull, Gumbel Max and Frechet distributions were estimated as shown on Table 2. Preliminary analysis on the data shows Weibull as the best fitting distribution for the data gathered, which is a three parameter distribution function. However, in measuring the risk associated in insuring such events, a threshold is needed in order to measure the exceedance of financial losses to set the value at risk as well as the expected shortfall for such insurance coverage.

TABLE 2. Probabilistic Distribution Fitting for Financial Loss Data

Distribution/Parameter	Shape	Scale	Location
Weibull	$0.41208 \times 10^6$	$5.8153 \times 10^5$	46
Gumbel Max	$8.0739 \times 10^6$	$-2.4101 \times 10^6$	-
Frechet	0.4554	40073	-

4.2. Thresholds and Estimating of Tail Distribution.

Due to the extremities in the data, a considerable number is suitable as an extreme enough to be chosen as a threshold, in this study the high quantile is chosen as the

threshold. By observation, a significant number of the data points were below GHS 3,003,400 which is around the 90th quantile and very few above this data point. Preliminary analysis of the rest of the data shows it follows Pareto tail behavior and a confirmation was conducted on Q-Q plot. The data above the threshold was fitted to Generalized Pareto Distribution (GPD) (Kolmogorov Smirnov Test P-value = 0.61) and the parameters of the distribution was determined with Maximum Likelihood Estimation method. The GPD parameters were found to be 0.4621 for shape parameter and 40271 for scale parameter. The normality of the fitted distributions were assessed with the Quantile-Quantile plot as shown in Figure 1, This is use to confirm the Paretian tail behavior of plots, the quantiles of the data set to the quantiles of the Generalized Pareto distribution were realized, it is obvious, the quantile distributions mostly occupy the reference line are found in both plots. After choosing the threshold, there were  $N_u = 23$  exceedance out of the

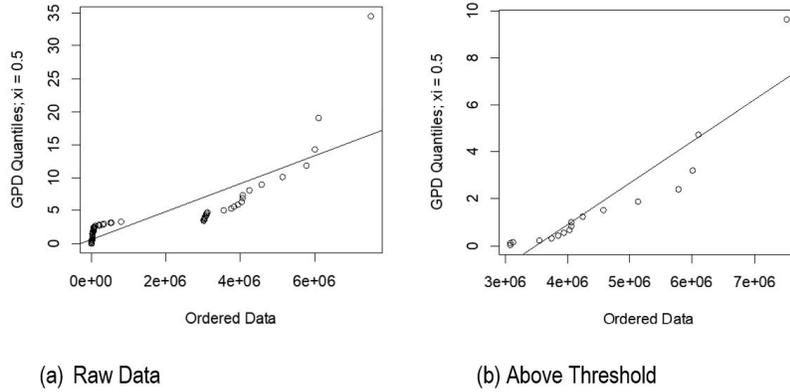


FIGURE 1. Quantile-Quantile Plot for Raw Data and Data Above threshold

$n = 167$  observations, the estimator of the tail behavior distribution function over the threshold  $\mu = 3,003,400$  as shown in Figure 3 which compares the exceedance plots for the data and after the threshold and its estimation is a function given as:

$$\hat{F} = 1 - \frac{23}{167} \left( 1 + \gamma \frac{x - 3003400}{\sigma} \right)^{-1/\gamma}$$

With shape and scale parameters of the GPD, the distribution function is then simplified to be

$$\hat{F} = 1 - \frac{23}{167} \left( 1 + 0.4621 \frac{x - 3003400}{40271} \right)^{-1/0.4621}$$

$$\hat{F} = 0.863 \left( 1 + 0.4621 \frac{x - 3003400}{40271} \right)^{-2.164}$$

This is only valid for observation exceeding the threshold thus  $P(X < x | X > U)$ . Using the tail behavior distribution function, similar function is estimated for the probability density function for particular observations in the data given us

$$\hat{f} = \frac{23}{167\sigma} \left( 1 + \gamma \frac{x - 3003400}{\sigma} \right)^{-1-1/\gamma} = P(X < x | X > \mu)$$

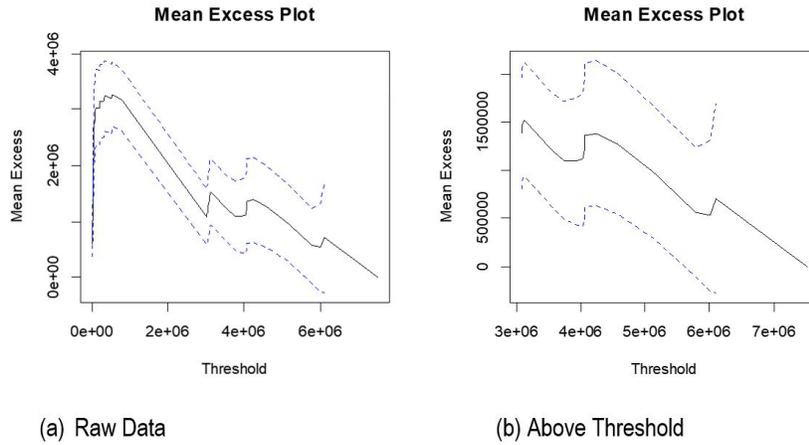


FIGURE 2. Exceedance Plot for Raw Data and Data Above threshold

$$= \frac{23}{167(40271)} \left( 1 + (0.4621) \frac{x - 3003400}{\sigma} \right)^{-3.164}$$

These results serve as the foundation for predictions of the various functional probabilities of extremities given any financial loss data due to fire outbreak in Ghana. Such refinement of a possible probability distribution function for modeling the losses might improve accuracy of extreme value estimates for a future forecast which will help in planning for mitigation of economic loss when such eventuality happens.

**4.3. Value at Risk and Expected Shortfalls.**

Based on the results obtained from equation (24) the Value at Risk is estimated for various quantiles as well as their corresponding Expected shortfalls as shown in Table 3. Notably, the 99th percentile shortfall is in excess of 28.89 million cedis. This information turns to give much preparedness for insurance companies to either beefed up their financial muscle or engage in reinsurance of facilities and other related policies which exposed to fire outbreak insurance.

TABLE 3. Value at Risk and Expected Shortfall

Quantile	0.90	0.95	0.98	0.99
Value at Risk (GHS)	30,047,292	30,085,439	30,158,815	30,239,067
Expected Shortfall (GHS)	28,699,691	28,737,838	28,811,214	28,891,466

**4.4. Return Level.**

The  $\delta$  year of return is the level of exceeded on average only once in  $\delta$  years, the return level for 2, 5, 10, 20, and 50 years have been estimated in Table 4. The result shows a consistent increasing of financial loss over the 50 year period which stood at GHS 30,198,807 in 5 years and in excess of and will exceed GHS 30,678,160 in 50 years, other words it is the least maximum extreme financial loss which will occur after 50 years.

TABLE 4. Return Level of Maximum financial loss (fire insurance claims)

Year	Return level				
	$\delta = 2$	$\delta = 5$	$\delta = 10$	$\delta = 20$	$\delta = 50$
Financial Loss (GHS)	30,111,627	30,198,807	30,294,159	30,425,509	30,425,509

#### 4.5. Discussion.

Insurance companies insuring fire outbreaks would have interesting tools at their disposal with the return levels and the value at risk as well as the expected shortfall results. These allow for a proper planning and preparation for period during which extremes are likely to occur. Practically, return levels are much easier to interpret than parameters and probability density functions (Coles et al., 2003). Additionally, the accuracy of the modeling procedures brings better financial predictions and hence influence insurance companies to make decisions on re-insurance to cover expected shortfalls. It is worthy to note, that return level indicates a level reached or exceeded once on average, over a long period of time. McNeils (1999) indicated a possibility of the return level being reached more than once or none at all. Though the EVT provides the needed tools for events with extremes, it is worth noting, it's also possess inherent uncertainty especially at small sample size. Nevertheless, the return level, expected shortfalls and value at risk provide useful information for better preparations (Robine et al, 2003). The shape parameter in the GEV and GPD provide new insights regarding peak distributions which serve as an upper bound interpreting as an existence of a maximum for which the financial losses cannot go beyond (Watts et al, 2006). These are essentials as there should be provision of occurrences of losses with extremely high severity in order to secure the future and keeps the economy running during these eventualities.

#### 4.6. Conclusion.

The study has illustrated how extreme value theory can be used to model tail-related risk measures such as Value-at-Risk, expected shortfall and return level, applying it to fire outbreak financial losses data. It has shown that EVT and most importantly the peaks over-threshold offers a good statistical tool to analyze high severities in extreme events, it is the most employed technique in risk management to deal with events where tails of probability distributions are considered. The results gathered give a good overview of financial losses as a result to fire outbreaks in Ghana especially within the markets where it affects the economy and incurs high severity losses. The return level predictions provide the need information for planning of future eventualities as well as for insurance coverage information for insurance companies.

#### List of Abbreviations.

**VaR:** Value at Risk  
**ES :** Expected Short-Fall  
**EVT:** Extreme Value Theory  
**GEV :** Generalized Extreme Value  
**GPD:** Generalized Pareto Distribution

## DECLARATIONS

**Availability of data and material.**

The data that support the findings of this study are available from Ghana National Fire Service Department and the National Disaster Management Organization, but restrictions apply to the availability of these data, which were used under license for the current study per conditions given for the released of data for the study, and so are not publicly available. Data are however available from the authors upon reasonable request and with permission of the Ghana National Fire Service Department and National Disaster Management Organization.

**Competing Interest.** The authors declare that they have no competing interests

**Funding.** The authors have no sources of funding towards this study

**Acknowledgments.** Not Applicable

**Authors Information.** Not Applicable

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